Decision 1: Travelling Salesman

Past Paper Questions
2006 - 2013

Name:

January 2006

8 Salvadore is visiting six famous places in Barcelona: La Pedrera (L), Nou Camp (N), Olympic Village (O), Park Guell (P), Ramblas (R) and Sagrada Familia (S). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel.

The table shows the times, in minutes, that it will take to travel between the six places.

From \ To	La Pedrera (L)	Nou Camp (N)	Olympic Village (O)	Park Guell (P)	Ramblas (R)	Sagrada Familia (S)
La Pedrera (L)	—	35	30	30	37	35
Nou Camp (N)	25	—	20	21	25	40
Olympic Village (O)	15	40	—	25	30	29
Park Guell (P)	30	35	25	—	35	20
Ramblas (R)	20	30	17	25	—	25
Sagrada Familia (S)	25	35	29	20	30	—

(a) Find the total travelling time for:

(i) the route $LNOL$; *(1 mark)*

(ii) the route $LONL$. *(1 mark)*

(b) Give an example of a Hamiltonian cycle in the context of the above situation. *(1 mark)*

(c) Salvadore intends to travel from one place to another until he has visited all of the places before returning to his starting place.

(i) Show that, using the nearest neighbour algorithm starting from Sagrada Familia (S), the total travelling time for Salvadore is 145 minutes. *(3 marks)*

(ii) Explain why your answer to part (c)(i) is an upper bound for the minimum travelling time for Salvadore. *(2 marks)*

(iii) Salvadore starts from Sagrada Familia (S) and then visits Ramblas (R). Given that he visits Nou Camp (N) before Park Guell (P), find an improved upper bound for the total travelling time for Salvadore. *(3 marks)*

5 (a) Gill is solving a travelling salesperson problem.

(i) She finds the following upper bounds: 7.5, 8, 7, 7.5, 8.5 .

Write down the best upper bound. *(1 mark)*

(ii) She finds the following lower bounds: 6.5, 7, 6.5, 5, 7.

Write down the best lower bound. *(1 mark)*

(b) George is travelling by plane to a number of cities. He is to start at F and visit each of the other cities at least once before returning to F.

The diagram shows the times of flights, in hours, between cities. Where no time is shown, there is no direct flight available.

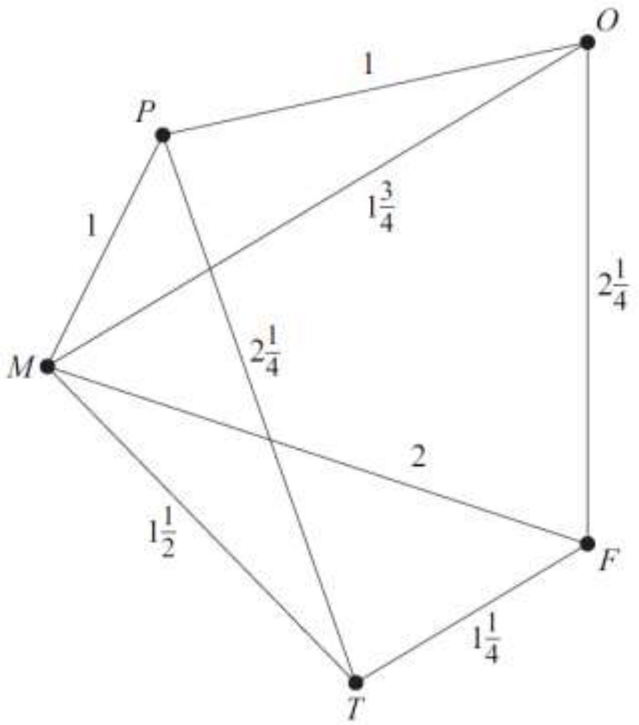


Figure 2

	M	P	O	T	F
M	–	1	$1\frac{3}{4}$	$1\frac{1}{2}$	2
P	1	–	1	$2\frac{1}{4}$	
O	$1\frac{3}{4}$	1	–		$2\frac{1}{4}$
T	$1\frac{1}{2}$	$2\frac{1}{4}$		–	$1\frac{1}{4}$
F	2		$2\frac{1}{4}$	$1\frac{1}{4}$	–

(i) Complete **Figure 2** to show the minimum times to travel between all pairs of cities. *(2 marks)*

(ii) Find an upper bound for the minimum total flying time by using the route *FTPOMF*. *(1 mark)*

(iii) Using the nearest neighbour algorithm starting from F, find an upper bound for the minimum total flying time. *(4 marks)*

(iv) By deleting F, find a lower bound for the minimum total flying time. *(5 marks)*

3 Mark is driving around the one-way system in Leicester. The following table shows the times, in minutes, for Mark to drive between four places: A, B, C and D. Mark decides to start from A, drive to the other three places and then return to A.

Mark wants to keep his driving time to a minimum.

From \ To	*A*	*B*	*C*	*D*
A	–	8	6	11
B	14	–	13	25
C	14	9	–	17
D	26	10	18	–

(a) Find the length of the tour $ABCDA$. *(2 marks)*

(b) Find the length of the tour $ADCBA$. *(1 mark)*

(c) Find the length of the tour using the nearest neighbour algorithm starting from A. *(4 marks)*

(d) Write down which of your answers to parts (a), (b) and (c) gives the best upper bound for Mark's driving time. *(1 mark)*

June 2007

6 (a) Mark is staying at the Grand Hotel (G) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge (A), the National Theatre (N), Parliament House (P) and the Royal Palace (R).

The figures in the table represent the walking times, in seconds, between the places.

	Grand Hotel (G)	**Aker Brygge (A)**	**National Theatre (N)**	**Parliament House (P)**	**Royal Palace (R)**
Grand Hotel (G)	–	165	185	65	160
Aker Brygge (A)	165	–	155	115	275
National Theatre (N)	185	155	–	205	125
Parliament House (P)	65	115	205	–	225
Royal Palace (R)	160	275	125	225	–

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.

(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour. *(4 marks)*

(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour. *(5 marks)*

(iii) The walking time for an optimal tour is T seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about T. *(1 mark)*

5 (a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

Write down the best upper bound. *(1 mark)*

(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound. *(1 mark)*

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. *(1 mark)*

(c) The diagram below shows roads connecting 4 towns, *A*, *B*, *C* and *D*. The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.

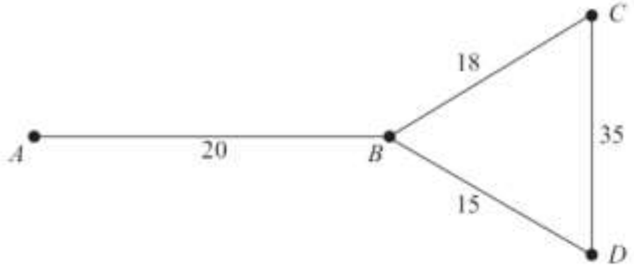


Figure 3

	A	*B*	*C*	*D*
A	—		38	
B		—		
C	38		—	
D				—

Xiong lives at town *A* and is to visit each of the other three towns before returning to town *A*. She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. *(2 marks)*

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at *A*. *(3 marks)*

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). *(2 marks)*

4 David, a tourist, wishes to visit five places in Rome: Basilica (*B*), Coliseum (*C*), Pantheon (*P*), Trevi Fountain (*T*) and Vatican (*V*). He is to start his tour at one of the places, visit each of the other places, before returning to his starting place.

The table shows the times, in minutes, to travel between these places. David wishes to keep his travelling time to a minimum.

	B	***C***	***P***	***T***	***V***
B	–	43	57	52	18
C	43	–	18	13	56
P	57	18	–	8	48
T	52	13	8	–	51
V	18	56	48	51	–

(a) (i) Find the total travelling time for the tour *TPVBCT*. *(1 mark)*

(ii) Find the total travelling time for David's tour using the nearest neighbour algorithm starting from *T*. *(4 marks)*

(iii) Explain why your answer to part (a)(ii) is an upper bound for David's minimum total travelling time. *(2 marks)*

(b) (i) By deleting *B*, find a lower bound for the total travelling time for the minimum tour. *(5 marks)*

(ii) Explain why your answer to part (b)(i) is a lower bound for David's minimum total travelling time. *(2 marks)*

(c) Sketch a network showing the edges that give the lower bound found in part (b)(i) and comment on its significance. *(2 marks)*

7 Liam is taking part in a treasure hunt. There are five clues to be solved and they are at the points A, B, C, D and E. The table shows the distances between pairs of points. All of the distances are functions of x, **where x is an integer**.

Liam must travel to all five points, starting and finishing at A.

	A	**B**	**C**	**D**	**E**
A	–	$x+6$	$2x-4$	$3x-7$	$4x-14$
B	$x+6$	–	$3x-7$	$3x-9$	$x+9$
C	$2x-4$	$3x-7$	–	$2x-1$	$x+8$
D	$3x-7$	$3x-9$	$2x-1$	–	$2x-2$
E	$4x-14$	$x+9$	$x+8$	$2x-2$	–

(a) The nearest point to A is C.

(i) By considering AC and AB, show that $x < 10$. *(2 marks)*

(ii) Find two other inequalities in x. *(2 marks)*

(b) The nearest neighbour algorithm, starting from A, gives a **unique** minimum tour $ACDEBA$.

(i) By considering the fact that Liam's tour visits D immediately after C, find two further inequalities in x. *(3 marks)*

(ii) Find the value of the integer x. *(4 marks)*

(iii) Hence find the total distance travelled by Liam if he uses this tour. *(2 marks)*

5 Angelo is visiting six famous places in Palermo: A, B, C, D, E and F. He intends to travel from one place to the next until he has visited all of the places before returning to his starting place. Due to the traffic system, the time taken to travel between two places may be different dependent on the direction travelled.

The table shows the times, in minutes, taken to travel between the six places.

From \ To	A	B	C	D	E	F
A	–	25	20	20	27	25
B	15	–	10	11	15	30
C	5	30	–	15	20	19
D	20	25	15	–	25	10
E	10	20	7	15	–	15
F	25	35	29	20	30	–

(a) Give an example of a Hamiltonian cycle in this context. *(2 marks)*

(b) (i) Show that, if the nearest neighbour algorithm starting from F is used, the total travelling time for Angelo would be 95 minutes. *(3 marks)*

(ii) Explain why your answer to part **(b)(i)** is an upper bound for the minimum travelling time for Angelo. *(2 marks)*

(c) Angelo starts from F and visits E next. He also visits B before he visits D. Find an improved upper bound for Angelo's total travelling time. *(3 marks)*

5 There is a one-way system in Manchester. Mia is parked at her base, B, in Manchester and intends to visit four other places, A, C, D and E, before returning to her base. The following table shows the distances, in kilometres, for Mia to drive between the five places A, B, C, D and E. Mia wants to keep the total distance that she drives to a minimum.

From \ To	A	B	C	D	E
A	–	1.7	1.9	1.8	2.1
B	3.1	–	2.5	1.8	3.7
C	3.1	2.9	–	2.7	4.2
D	2.0	2.8	2.1	–	2.3
E	2.2	3.6	1.9	1.7	–

(a) Find the length of the tour $BECDAB$. *(1 mark)*

(b) Find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

(c) Write down which of your answers to parts (a) and (b) would be the better upper bound for the total distance that Mia drives. *(1 mark)*

(d) On a particular day, the council decides to reverse the one-way system. For this day, find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

5 Phil, a squash coach, wishes to buy some equipment for his club. In a town centre, there are six shops, G, I, N, R, S and T, that sell the equipment.

The time, in seconds, to walk between each pair of shops is shown in the table.

Phil intends to check prices by visiting each of the six shops before returning to his starting point.

	G	***I***	***N***	***R***	***S***	***T***
G	–	81	82	86	72	76
I	81	–	80	82	68	73
N	82	80	–	84	70	74
R	86	82	84	–	74	70
S	72	68	70	74	–	64
T	76	73	74	70	64	–

(a) Use the nearest neighbour algorithm starting from S to find an upper bound for Phil's minimum walking time. *(4 marks)*

(b) Write down a tour starting from N which has a total walking time equal to your answer to part **(a)**. *(1 mark)*

(c) By deleting S, find a lower bound for Phil's minimum walking time. *(5 marks)*

January 2011

7 Fred delivers bread to five shops, A, B, C, D and E. Fred starts his deliveries at shop B, and travels to each of the other shops once before returning to shop B. Fred wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	***B***	***C***	***D***	***E***
A	–	3	11	15	5
B	3	–	18	12	4
C	11	18	–	5	16
D	15	12	5	–	10
E	5	4	16	10	–

(a) Find the travelling time for the tour $BACDEB$. *(1 mark)*

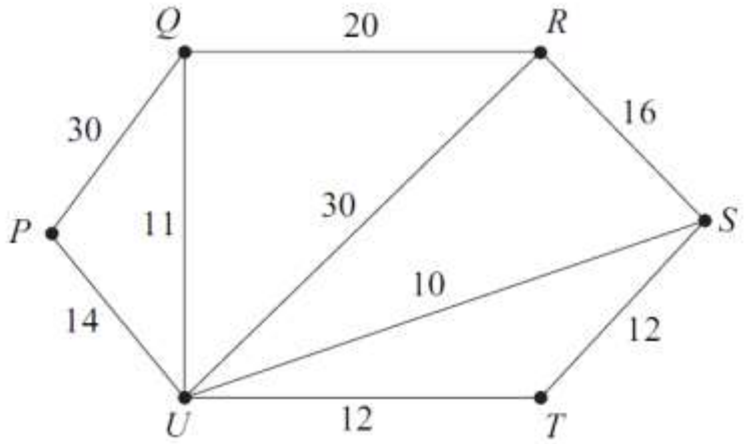
(b) Use the nearest neighbour algorithm, starting at B, to find another upper bound for the travelling time for Fred's tour. *(3 marks)*

(c) By deleting C, find a lower bound for the travelling time for Fred's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

8 Mrs Jones is a spy who has to visit six locations, P, Q, R, S, T and U, to collect information. She starts at location Q, and travels to each of the other locations, before returning to Q. She wishes to keep her travelling time to a minimum.

The diagram represents roads connecting different locations. The number on each edge represents the travelling time, in minutes, along that road.



(a) (i) Explain why the shortest time to travel from P to R is 40 minutes. *(2 marks)*

(ii) Complete **Table 1**, on the opposite page, in which the entries are the shortest travelling times, in minutes, between pairs of locations. *(2 marks)*

(b) (i) Use the nearest neighbour algorithm on **Table 1**, starting at Q, to find an upper bound for the minimum travelling time for Mrs Jones's tour. *(4 marks)*

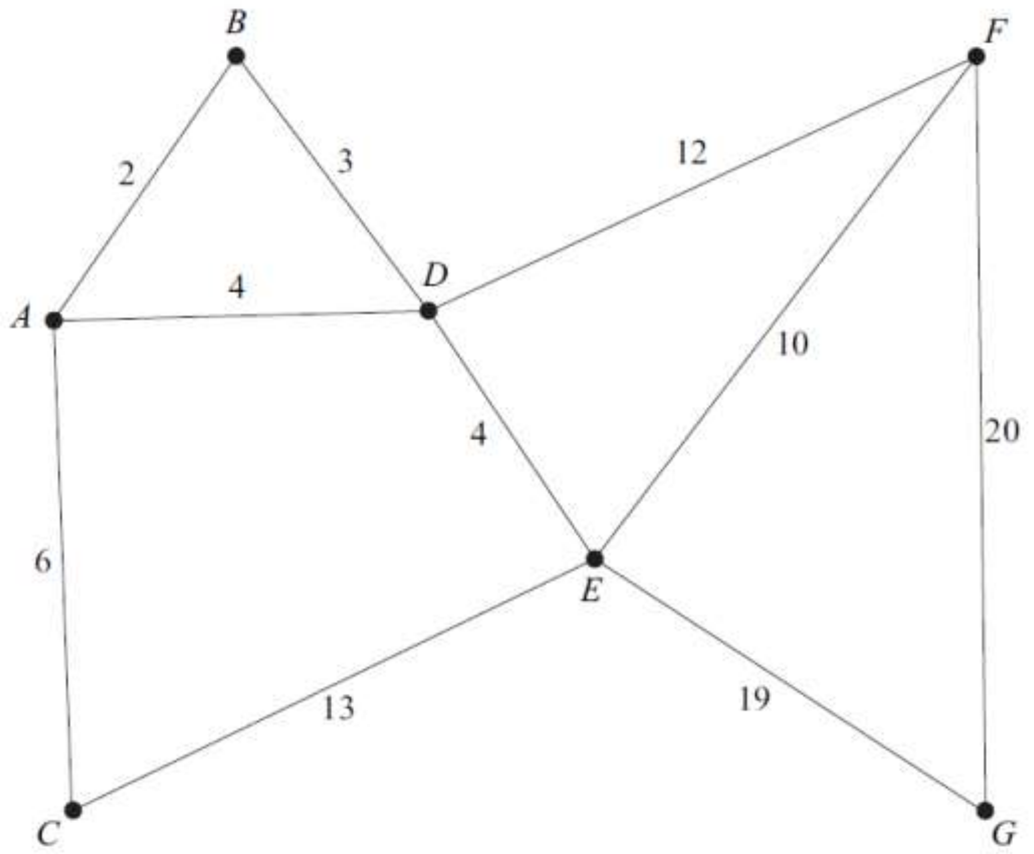
(ii) Mrs Jones decides to follow the route given by the nearest neighbour algorithm. Write down her route, showing all the locations through which she passes. *(2 marks)*

(c) By deleting Q from **Table 1**, find a lower bound for the travelling time for Mrs Jones's tour. *(5 marks)*

Table 1

	P	***Q***	***R***	***S***	***T***	***U***
P	–	25				
Q	25	–	20	21	23	11
R		20	–			
S		21		–		
T		23			–	12
U		11			12	–

7 The diagram shows the locations of some schools. The number on each edge shows the distance, in miles, between pairs of schools.



Sam, an adviser, intends to travel from one school to the next until he has visited all of the schools, before returning to his starting school. The shortest distances for Sam to travel between some of the schools are shown in **Table 1** opposite.

(a) Complete **Table 1**. *(2 marks)*

(b) (i) On the completed **Table 1**, use the nearest neighbour algorithm, starting from *B*, to find an upper bound for the length of Sam's tour. *(4 marks)*

(ii) Write down Sam's actual route if he were to follow the tour corresponding to the answer in part **(b)(i)**. *(2 marks)*

(iii) Using the nearest neighbour algorithm, starting from each of the other vertices in turn, the following upper bounds for the length of Sam's tour were obtained: 77, 77, 77, 76, 77 and 76.

Write down the best upper bound. *(1 mark)*

Table 1

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	2	6	4		16	27
B	2	–	8	3		15	26
C	6	8	–	10		22	32
D	4	3	10	–		12	23
E					–		
F	16	15	22	12		–	20
G	27	26	32	23		20	–

7 (c) (i) On **Table 2** below, showing the order in which you select the edges, use Prim's algorithm, starting from *A*, to find a minimum spanning tree for the schools *A*, *B*, *C*, *D*, *F* and *G*. *(4 marks)*

(ii) Hence find a lower bound for the length of Sam's minimum tour. *(3 marks)*

(iii) By deleting each of the other vertices in turn, the following lower bounds for the length of a minimum tour were found: 50, 48, 52, 51, 56 and 64.

Write down the best lower bound. *(1 mark)*

(d) Given that the length of a minimum tour is T miles, use your answers to parts **(b)** and **(c)** to write down the smallest interval within which you know T must lie. *(2 marks)*

Table 2

	A	*B*	*C*	*D*	*F*	*G*
A	–	2	6	4	16	27
B	2	–	8	3	15	26
C	6	8	–	10	22	32
D	4	3	10	–	12	23
F	16	15	22	12	–	20
G	27	26	32	23	20	–

June 2012

7 Rupta, a sales representative, has to visit six shops, A, B, C, D, E and F. Rupta starts at shop A and travels to each of the other shops once, before returning to shop A. Rupta wishes to keep her travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	***B***	***C***	***D***	***E***	***F***
A	–	16	10	25	26	40
B	16	–	20	19	18	50
C	10	20	–	14	22	31
D	25	19	14	–	11	32
E	26	18	22	11	–	42
F	40	50	31	32	42	–

(a) Find the travelling time of the tour $ACFDEBA$. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the travelling time for Rupta's tour. *(4 marks)*

(c) By deleting A, find a lower bound for the travelling time for Rupta's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

8 Tony delivers paper to five offices, A, B, C, D and E. Tony starts his deliveries at office E and travels to each of the other offices once, before returning to office E. Tony wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the offices.

	A	***B***	***C***	***D***	***E***
A	–	10	16	20	8
B	10	–	21	15	9
C	16	21	–	10	23
D	20	15	10	–	17
E	8	9	23	17	–

(a) Find the travelling time of the tour $ACDBEA$. *(1 mark)*

(b) Hence write down a tour, starting at E, which has the same total travelling time as your answer to part **(a)**. *(1 mark)*

(c) Use the nearest neighbour algorithm, starting at E, to find an upper bound for the minimum travelling time for Tony's tour. *(4 marks)*

(d) By deleting E, find a lower bound for the minimum travelling time for Tony's tour. *(4 marks)*

(e) Sketch a network showing the edges that give the lower bound in part **(d)**, and comment on its significance. *(2 marks)*

4 Sarah is a mobile hairdresser based at A. Her day's appointments are at five places: B, C, D, E and F. She can arrange the appointments in any order. She intends to travel from one place to the next until she has visited all of the places, starting and finishing at A. The following table shows the times, in minutes, that it takes to travel between the six places.

	A	***B***	***C***	***D***	***E***	***F***
A	–	15	11	14	27	12
B	15	–	13	19	24	15
C	11	13	–	10	19	12
D	14	19	10	–	26	15
E	27	24	19	26	–	27
F	12	15	12	15	27	–

(a) Sarah decides to visit the places in the order $ABCDEFA$. Find the travelling time of this tour. *(1 mark)*

(b) Explain why this answer can be considered as being an upper bound for the minimum travelling time of Sarah's tour. *(2 marks)*

(c) Use the nearest neighbour algorithm, starting from A, to find another upper bound for the minimum travelling time of Sarah's tour. *(4 marks)*

(d) By deleting A, find a lower bound for the minimum travelling time of Sarah's tour. *(4 marks)*

(e) Sarah thinks that she can reduce her travelling time to 75 minutes. Explain why she is wrong. *(1 mark)*