Core 3: Trigonometry

Past Paper Questions
2006 - 2013

Name:

January 2006

4 It is given that $2\text{cosec}^2 x = 5 - 5\cot x$.

(a) Show that the equation $2\text{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\text{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

June 2006

3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3\sec x + 9$ can be written as

$$\sec^2 x - 3\sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3\sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

January 2007

5 (a) (i) Show that the equation

$$2\cot^2 x + 5\,\text{cosec}\,x = 10$$

can be written in the form $2\,\text{cosec}^2 x + 5\,\text{cosec}\,x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

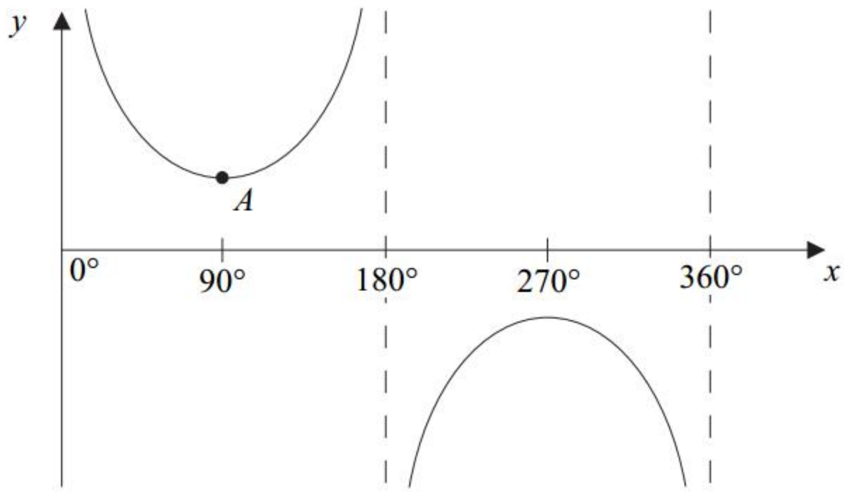
$$2\cot^2(\theta - 0.1) + 5\,\text{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

June 2007

3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$. *(2 marks)*

(b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0° < x < 360°$.



(i) The point A on the curve is where $x = 90°$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0° < x < 360°$. *(2 marks)*

(c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0° < x < 360°$. *(2 marks)*

8 (c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

January 2008

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(2 marks)*

(b) Show that the equation $\operatorname{cosec}^2 x = \dfrac{3\cot x + 4}{2}$ can be written as

$$2\cot^2 x - 3\cot x - 2 = 0$$

(2 marks)

(c) Solve the equation $\operatorname{cosec}^2 x = \dfrac{3\cot x + 4}{2}$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

June 2008

2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(3 marks)*

(b) Show that the equation $\tan^2 x = 2\sec x + 2$ can be written as $\sec^2 x - 2\sec x - 3 = 0$. *(2 marks)*

(c) Solve the equation $\tan^2 x = 2\sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(4 marks)*

January 2009

4 (a) Solve the equation $\sec x = \frac{3}{2}$, giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(2 marks)*

(b) By using a suitable trigonometrical identity, solve the equation

$$2\tan^2 x = 10 - 5\sec x$$

giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(6 marks)*

June 2009

3 (a) Solve the equation $\tan x = -\frac{1}{3}$, giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation

$$3\sec^2 x = 5(\tan x + 1)$$

can be written in the form $3\tan^2 x - 5\tan x - 2 = 0$. *(1 mark)*

(c) Hence, or otherwise, solve the equation

$$3\sec^2 x = 5(\tan x + 1)$$

giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(4 marks)*

January 2010

3 (a) Solve the equation

$$\operatorname{cosec} x = 3$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$.

(2 marks)

(b) By using a suitable trigonometric identity, solve the equation

$$\cot^2 x = 11 - \operatorname{cosec} x$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$.

(6 marks)

June 2010

5 (a) Show that the equation

$$10 \operatorname{cosec}^2 x = 16 - 11 \cot x$$

can be written in the form

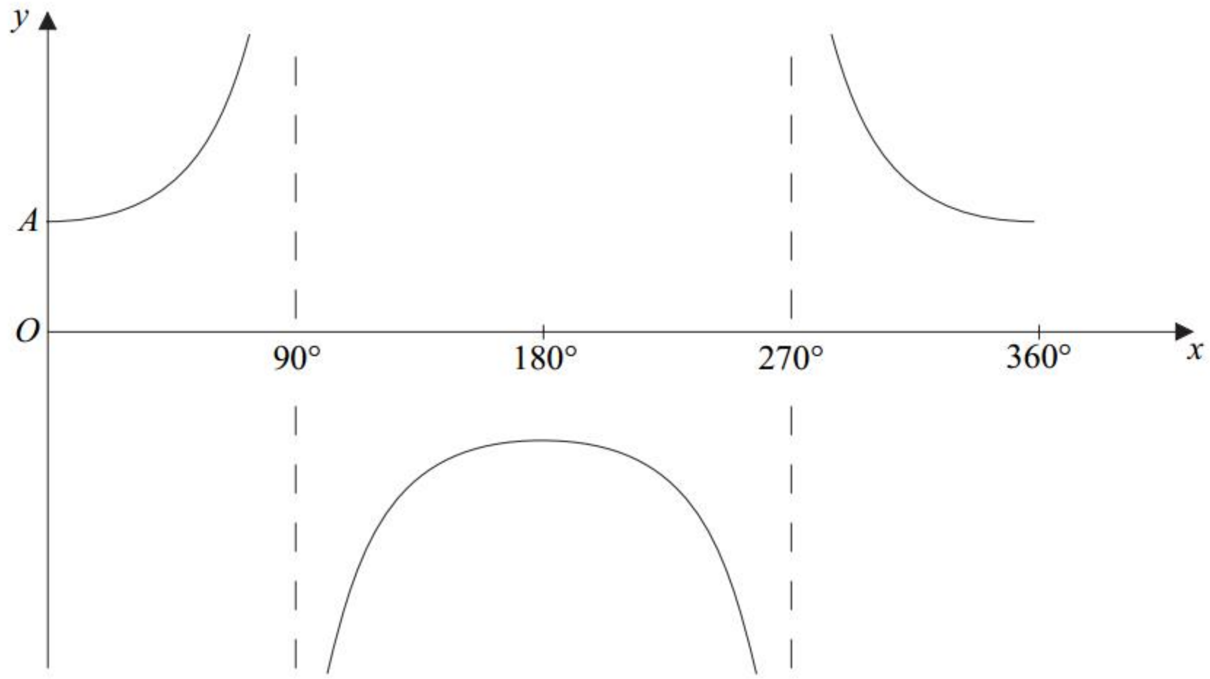
$$10 \cot^2 x + 11 \cot x - 6 = 0$$

(1 mark)

(b) Hence, given that $10 \operatorname{cosec}^2 x = 16 - 11 \cot x$, find the possible values of $\tan x$.

(4 marks)

2 (a) The diagram shows the graph of $y = \sec x$ for $0° \leqslant x \leqslant 360°$.

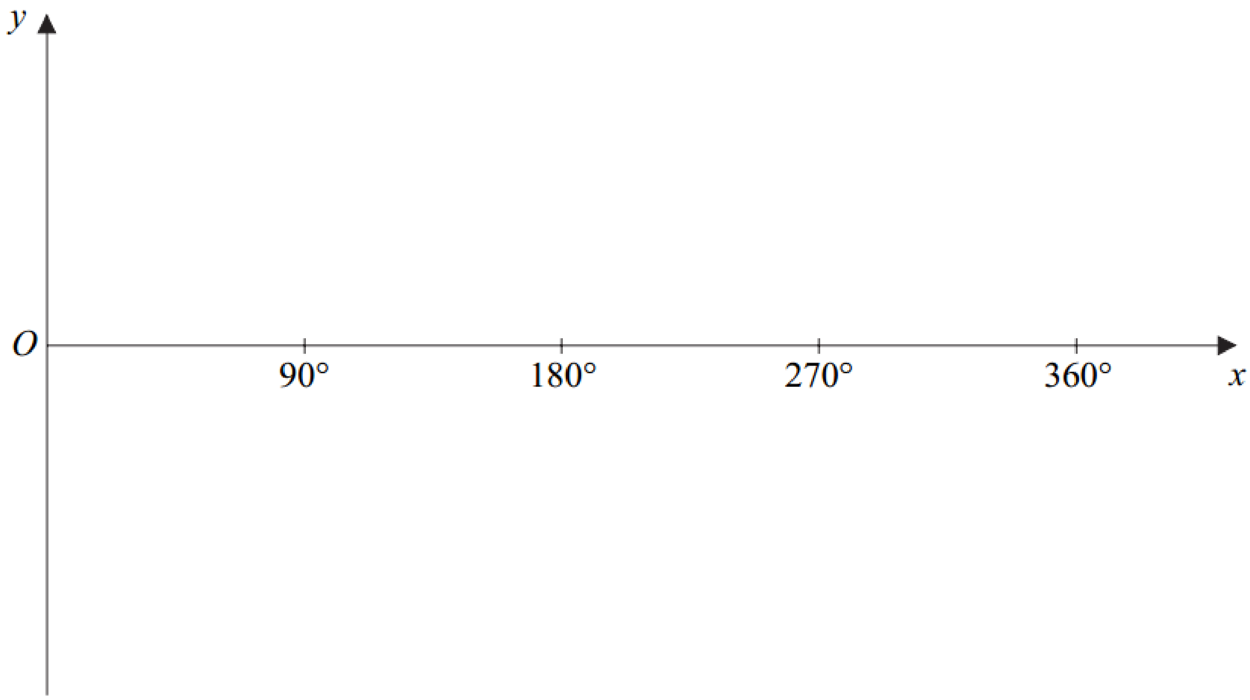


(i) The point A on the curve is where $x = 0$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch, on the axes given on page **5**, the graph of $y = |\sec 2x|$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) Solve the equation $|\sec(2x - 10°)| = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*



January 2011

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25$$ *(4 marks)*

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

June 2011

4 (a) (i) Solve the equation $\operatorname{cosec} \theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°. *(2 marks)*

(ii) Solve the equation

$$2\cot^2(2x + 30°) = 2 - 7\operatorname{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°. *(6 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \operatorname{cosec} x$ onto the graph of $y = \operatorname{cosec}(2x + 30°)$. *(4 marks)*

January 2012

4 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2 \theta = 3(3 - \sec\theta)$$

giving all solutions to the nearest 0.1° in the interval $0° < \theta < 360°$. *(6 marks)*

(b) Hence solve the equation

$$\tan^2(4x - 10°) = 3[3 - \sec(4x - 10°)]$$

giving all solutions to the nearest 0.1° in the interval $0° < x < 90°$. *(3 marks)*

June 2012

8 (a) Show that the equation

$$\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=32$$

can be written in the form

$$\operatorname{cosec}^2\theta = 16$$

(4 marks)

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1+\cos(2x-0.6)}+\frac{1}{1-\cos(2x-0.6)}=32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.

(5 marks)

January 2013

6 (a) Show that

$$\frac{\sec^2 x}{(\sec x+1)(\sec x-1)}$$

can be written as $\operatorname{cosec}^2 x$. *(3 marks)*

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x+1)(\sec x-1)}=\operatorname{cosec} x+3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$.

(6 marks)

(c) Hence solve the equation

$$\frac{\sec^2(2\theta-60°)}{(\sec(2\theta-60°)+1)(\sec(2\theta-60°)-1)}=\operatorname{cosec}(2\theta-60°)+3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$. *(2 marks)*

June 2013

4 By forming and solving a quadratic equation, solve the equation

$$8\sec x-2\sec^2 x=\tan^2 x-2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures.

(7 marks)