Core 2: Trigonometry

Past Paper Questions
2006 - 2013

Name:

Trigonometry – the Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

January 2006

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2\sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

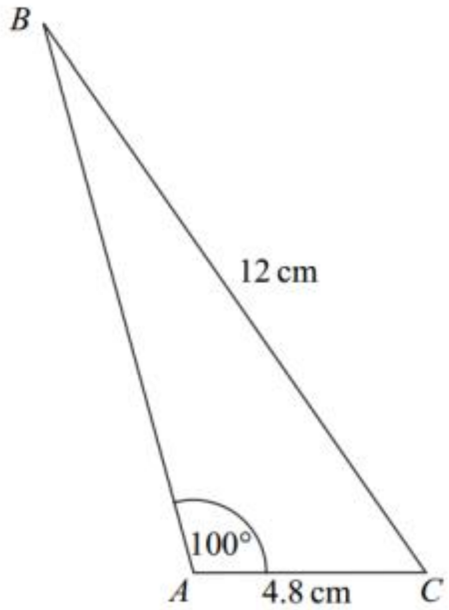
(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest $0.1°$ in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

June 2006

2 The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan\frac{1}{2}x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(b) Solve the equation $\tan\frac{1}{2}x = 3$ in the interval $\mathbf{0 < x < 4\pi}$, giving your answers in radians to three significant figures. *(4 marks)*

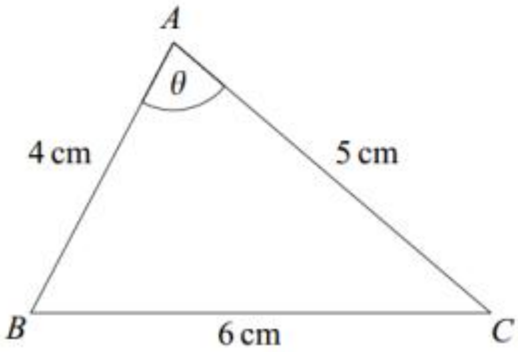
(c) Solve the equation

$$\cos\theta(\sin\theta - 3\cos\theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. *(5 marks)*

January 2007

4 The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.



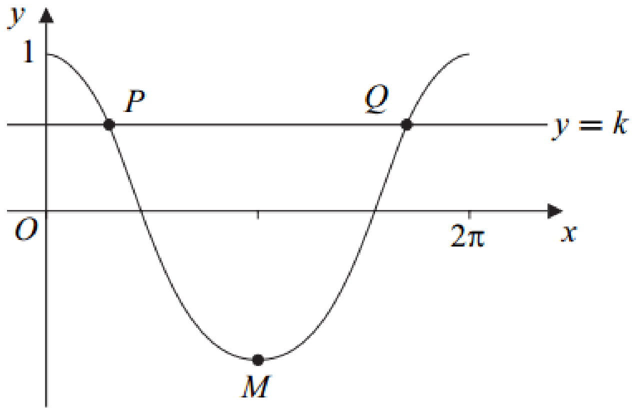
(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \dfrac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos \dfrac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

June 2007

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

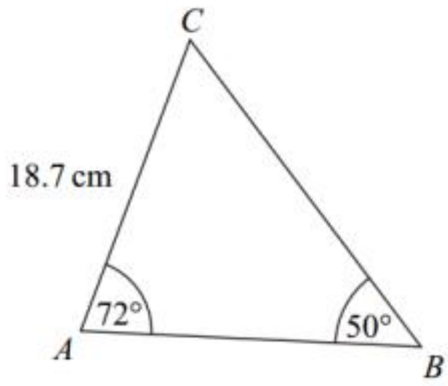
(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

January 2008

3 The diagram shows a triangle ABC. The length of AC is 18.7 cm, and the sizes of angles BAC and ABC are 72° and 50° respectively.



(a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest cm^2. *(3 marks)*

9 (a) Given that

$$\frac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta$$

show that

$$\cos\theta = -\frac{1}{2}$$ *(4 marks)*

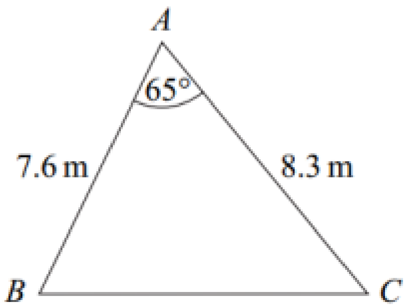
(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3\cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

June 2008

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

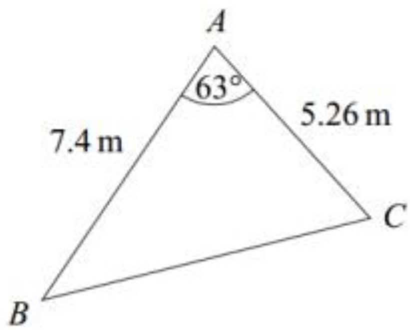
Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

9 (a) Solve the equation $\sin 2x = \sin 48°$, giving the values of x in the interval $0° \leqslant x < 360°$. *(4 marks)*

(b) Solve the equation $2\sin\theta - 3\cos\theta = 0$ in the interval $0° \leqslant \theta < 360°$, giving your answers to the nearest 0.1°. *(4 marks)*

January 2009

3 The diagram shows a triangle ABC.



The size of angle A is 63°, and the lengths of AB and AC are 7.4 m and 5.26 m respectively.

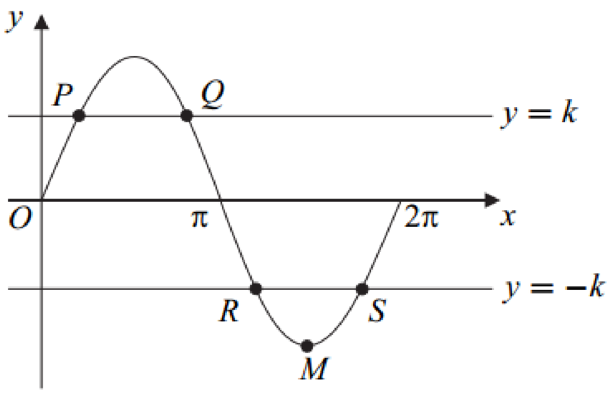
(a) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(b) Show that the length of BC is 6.86 m, correct to three significant figures. *(3 marks)*

(c) Find the value of **sin *B*** to two significant figures. *(2 marks)*

7 (a) Solve the equation $\sin x = 0.8$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of the curve $y = \sin x$, $0 \leqslant x \leqslant 2\pi$ and the lines $y = k$ and $y = -k$.



The line $y = k$ intersects the curve at the points P and Q, and the line $y = -k$ intersects the curve at the points R and S.

The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

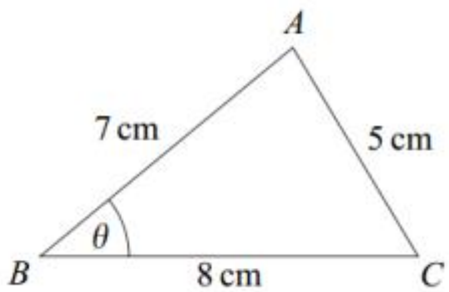
Write down the x-coordinate of the point Q in terms of π and α. *(1 mark)*

(iii) Find the length of RS in terms of π and α, giving your answer in its simplest form. *(2 marks)*

(c) Sketch the graph of $y = \sin 2x$ for $0 \leqslant x \leqslant 2\pi$, indicating the coordinates of points where the graph intersects the x-axis and the coordinates of any maximum points. *(5 marks)*

June 2009

1 The triangle ABC, shown in the diagram, is such that $AB = 7\text{ cm}$, $AC = 5\text{ cm}$, $BC = 8\text{ cm}$ and angle $ABC = \theta$.



(a) Show that $\theta = 38.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

8 (a) Given that $\dfrac{\sin\theta - \cos\theta}{\cos\theta} = 4$, prove that $\tan\theta = 5$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$2\cos^2 x - \sin x = 1$$

can be written as

$$2\sin^2 x + \sin x - 1 = 0$$ *(2 marks)*

(ii) Hence solve the equation

$$2\cos^2 x - \sin x = 1$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. *(5 marks)*

January 2010

8 (a) Solve the equation $\tan(x + 52°) = \tan 22°$, giving the values of x in the interval $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) (i) Show that the equation

$$3\tan\theta = \frac{8}{\sin\theta}$$

can be written as

$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(3 marks)*

(ii) Find the value of $\cos\theta$ that satisfies the equation

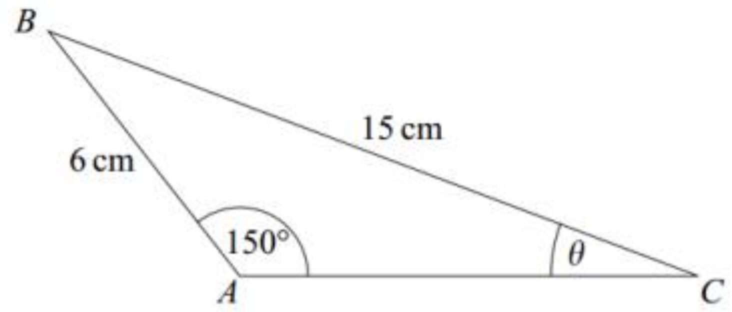
$$3\cos^2\theta + 8\cos\theta - 3 = 0$$ *(2 marks)*

(iii) Hence solve the equation

$$3\tan 2x = \frac{8}{\sin 2x}$$

giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*

3 The triangle ABC, shown in the diagram, is such that $AB = 6\text{ cm}$, $BC = 15\text{ cm}$, angle $BAC = 150°$ and angle $ACB = \theta$.



(a) Show that $\theta = 11.5°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

7 (a) Sketch the graph of $y = \cos x$ in the interval $0 \leqslant x \leqslant 2\pi$. State the values of the intercepts with the coordinate axes. *(2 marks)*

(b) (i) Given that

$$\sin^2 \theta = \cos \theta (2 - \cos \theta)$$

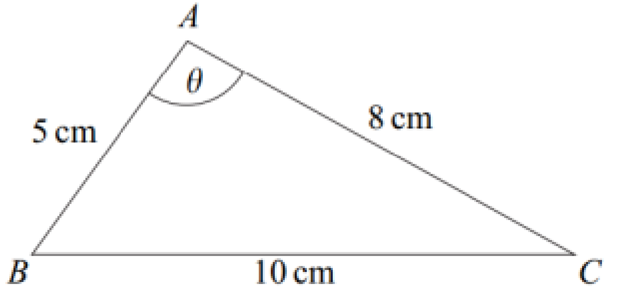
prove that $\cos \theta = \frac{1}{2}$. *(2 marks)*

(ii) Hence solve the equation

$$\sin^2 2x = \cos 2x(2 - \cos 2x)$$

in the interval $0 \leqslant x \leqslant \pi$, giving your answers in radians to three significant figures. *(4 marks)*

3 The triangle ABC, shown in the diagram, is such that $AB = 5\text{ cm}$, $AC = 8\text{ cm}$, $BC = 10\text{ cm}$ and angle $BAC = \theta$.



(a) Show that $\theta = 97.9°$, correct to the nearest $0.1°$. *(3 marks)*

(b) (i) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

(ii) The line through A, perpendicular to BC, meets BC at the point D. Calculate the length of AD, giving your answer, in cm, to three significant figures. *(3 marks)*

9 (a) Solve the equation $\tan x = -3$ in the interval $0° \leqslant x \leqslant 360°$, giving your answers to the nearest degree. *(3 marks)*

(b) (i) Given that

$$7\sin^2\theta + \sin\theta\cos\theta = 6$$

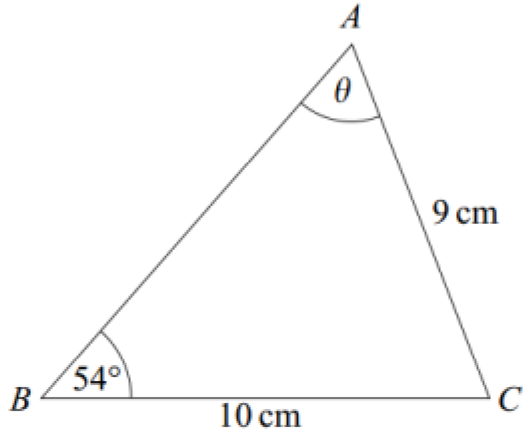
show that

$$\tan^2\theta + \tan\theta - 6 = 0$$ *(3 marks)*

(ii) Hence solve the equation $7\sin^2\theta + \sin\theta\cos\theta = 6$ in the interval $0° \leqslant \theta \leqslant 360°$, giving your answers to the nearest degree. *(4 marks)*

June 2011

1 The triangle ABC, shown in the diagram, is such that $AC = 9\text{ cm}$, $BC = 10\text{ cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.



(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

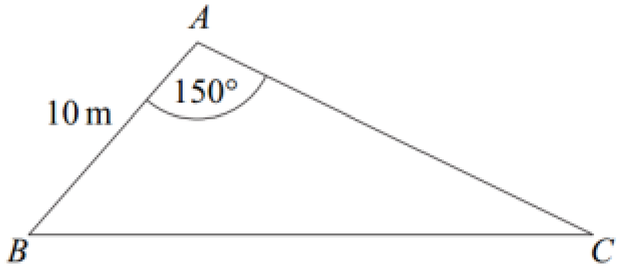
8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

January 2012

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is 40 m^2.

(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$6\sin^2 x = 4 + \cos x$$

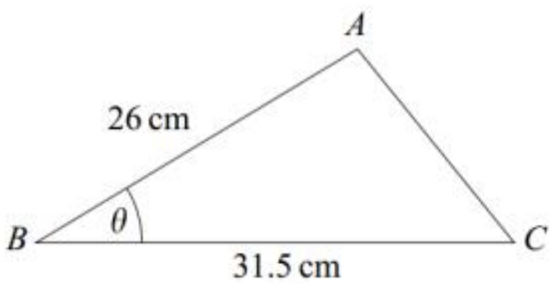
can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

June 2012

2 The triangle ABC, shown in the diagram, is such that $AB = 26$ cm and $BC = 31.5$ cm.



The acute angle ABC is θ, where $\sin\theta = \frac{5}{13}$.

(a) Calculate the area of triangle ABC. *(2 marks)*

(b) Find the exact value of $\cos\theta$. *(1 mark)*

(c) Calculate the length of AC. *(3 marks)*

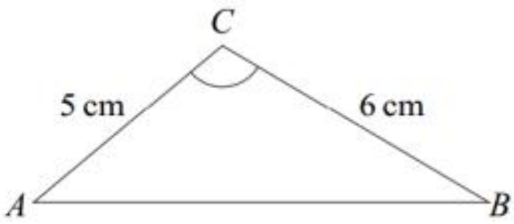
7 It is given that $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$.

(a) Find the possible values of $\tan\theta$. *(4 marks)*

(b) Hence solve the equation $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$, giving all solutions for θ, in degrees, in the interval $0° \leqslant \theta \leqslant 180°$. *(3 marks)*

January 2013

3 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is $12.5\ \text{cm}^2$, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest $0.1°$. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

9 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$. *(2 marks)*

(c) (i) Given that $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$. *(5 marks)*

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures. *(3 marks)*