Core 4: Vectors

Past Exam Questions
2006 - 2013

Name:

January 2006

7 The quadrilateral $ABCD$ has vertices $A(2,1,3)$, $B(6,5,3)$, $C(6,1,-1)$ and $D(2,-3,-1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that the line AB is parallel to l_1. *(1 mark)*

(iii) Verify that D lies on l_1. *(2 marks)*

(b) The line l_2 passes through $D(2,-3,-1)$ and $M(4,1,1)$.

(i) Find the vector equation of l_2. *(2 marks)*

(ii) Find the angle between l_2 and AC. *(3 marks)*

June 2006

6 The points A and B have coordinates $(2, 4, 1)$ and $(3, 2, -1)$ respectively. The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$, where O is the origin.

(a) Find the vectors:

(i) $\overrightarrow{OC}$; *(1 mark)*

(ii) $\overrightarrow{AB}$. *(2 marks)*

(b) (i) Show that the distance between the points A and C is 5. *(2 marks)*

(ii) Find the size of angle BAC, giving your answer to the nearest degree. *(4 marks)*

(c) The point $P(\alpha, \beta, \gamma)$ is such that BP is perpendicular to AC.

Show that $4\alpha - 3\gamma = 15$. *(3 marks)*

January 2007

6 The points A, B and C have coordinates $(3, -2, 4)$, $(5, 4, 0)$ and $(11, 6, -4)$ respectively.

(a) (i) Find the vector $\overrightarrow{BA}$. *(2 marks)*

(ii) Show that the size of angle ABC is $\cos^{-1}\left(-\frac{5}{7}\right)$. *(5 marks)*

(b) The line l has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.

(i) Verify that C lies on l. *(2 marks)*

(ii) Show that AB is parallel to l. *(1 mark)*

(c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D. *(3 marks)*

June 2007

7 The lines l_1 and l_2 have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that l_1 and l_2 are perpendicular. *(2 marks)*

(b) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection, P. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on l_2. The point B on l_1 is such that $AP = BP$.

Find the length of AB. *(4 marks)*

January 2008

9 The points A and B lie on the line l_1 and have coordinates $(2, 5, 1)$ and $(4, 1, -2)$ respectively.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find a vector equation of the line l_1, with parameter λ. *(1 mark)*

(b) The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

(i) Show that the point $P(-2, -3, 5)$ lies on l_2. *(2 marks)*

(ii) The point Q lies on l_1 and is such that PQ is perpendicular to l_2. Find the coordinates of Q. *(6 marks)*

June 2008

7 The coordinates of the points A and B are $(3, -2, 1)$ and $(5, 3, 0)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$.

(a) Find the distance between A and B. *(2 marks)*

(b) Find the acute angle between the lines AB and l. Give your answer to the nearest degree. *(5 marks)*

(c) The points B and C lie on l such that the distance AC is equal to the distance AB. Find the coordinates of C. *(5 marks)*

January 2009

8 The points A and B have coordinates $(2, 1, -1)$ and $(3, 1, -2)$ respectively. The angle OBA is θ, where O is the origin.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Show that $\cos\theta = \dfrac{5}{2\sqrt{7}}$. *(4 marks)*

(b) The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OB}$. The line l is parallel to $\overrightarrow{AB}$ and passes through the point C. Find a vector equation of l. *(2 marks)*

(c) The point D lies on l such that angle $ODC = 90°$. Find the coordinates of D. *(4 marks)*

June 2009

7 The points A and B have coordinates $(3, -2, 5)$ and $(4, 0, 1)$ respectively.

The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$.

(a) Find the distance between the points A and B. *(2 marks)*

(b) Verify that B lies on l_1. *(2 marks)*

(c) The line l_2 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 3 \\ -8 \end{bmatrix}$.

The lines l_1 and l_2 intersect at the point C. Show that the points A, B and C form an isosceles triangle. *(6 marks)*

January 2010

8 The points A, B and C have coordinates $(2, -1, -5)$, $(0, 5, -9)$ and $(9, 2, 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$.

(a) Verify that the point B lies on the line l. *(2 marks)*

(b) Find the vector $\overrightarrow{BC}$. *(2 marks)*

(c) The point D is such that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

(i) Show that D has coordinates $(20, -7, 19)$. *(2 marks)*

(ii) The point P lies on l where $\lambda = p$. The line PD is perpendicular to l. Find the value of p. *(5 marks)*

7 The point A has coordinates $(4, -3, 2)$.

The line l_1 passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$.

The point B lies on l_2 where $\mu = 2$.

(a) Find the vector $\overrightarrow{AB}$. *(3 marks)*

(b) (i) Show that the lines l_1 and l_2 intersect. *(4 marks)*

(ii) The lines l_1 and l_2 intersect at the point P. Find the coordinates of P. *(1 mark)*

(c) The point C lies on a line which is parallel to l_1 and which passes through the point B. The points A, B, C and P are the vertices of a parallelogram.

Find the coordinates of the two possible positions of the point C. *(4 marks)*

8 The coordinates of the points A and B are $(3, -2, 4)$ and $(6, 0, 3)$ respectively.

The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

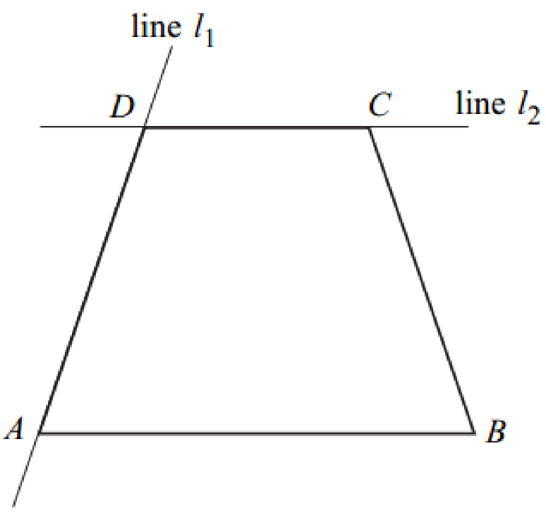
(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Calculate the acute angle between $\overrightarrow{AB}$ and the line l_1, giving your answer to the nearest 0.1°. *(4 marks)*

(b) The point D lies on l_1 where $\lambda = 2$. The line l_2 passes through D and is parallel to AB.

(i) Find a vector equation of line l_2 with parameter μ. *(2 marks)*

(ii) The diagram shows a symmetrical trapezium $ABCD$, with angle DAB equal to angle ABC.



The point C lies on line l_2. The length of AD is equal to the length of BC.

Find the coordinates of C. *(6 marks)*

June 2011

5 The points A and B have coordinates $(5,\ 1,\ -2)$ and $(4,\ -1,\ 3)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} -8 \\ 5 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$.

(a) Find a vector equation of the line that passes through A and B. *(3 marks)*

(b) (i) Show that the line that passes through A and B intersects the line l, and find the coordinates of the point of intersection, P. *(4 marks)*

(ii) The point C lies on l such that triangle PBC has a right angle at B. Find the coordinates of C. *(5 marks)*

January 2012

8 The points A and B have coordinates $(4,\ -2,\ 3)$ and $(2,\ 0,\ -1)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$.

(a) (i) Find the vector $\overrightarrow{AB}$. *(2 marks)*

(ii) Find the acute angle between AB and the line l, giving your answer to the nearest degree. *(4 marks)*

(b) The point C lies on the line l such that the angle ABC is a right angle. Given that $ABCD$ is a rectangle, find the coordinates of the point D. *(6 marks)*

June 2012

7 The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 0 \\ -2 \\ q \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, where q is an integer.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix}$.

The lines l_1 and l_2 intersect at the point P.

(a) Show that $q = 4$ and find the coordinates of P. *(3 marks)*

(b) Show that l_1 and l_2 are perpendicular. *(1 mark)*

(c) The point A lies on the line l_1 where $\lambda = 1$.

(i) Find AP^2. *(2 marks)*

(ii) The point B lies on the line l_2 so that the right-angled triangle APB is isosceles.

Find the coordinates of the two possible positions of B. *(6 marks)*

January 2013

6 (a) The points A, B and C have coordinates $(3, 1, -6)$, $(5, -2, 0)$ and $(8, -4, -6)$ respectively.

(i) Show that the vector $\overrightarrow{AC}$ is given by $\overrightarrow{AC} = n \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, where n is an integer. *(1 mark)*

(ii) Show that the acute angle ACB is given by $\cos^{-1}\left(\frac{5\sqrt{2}}{14}\right)$. *(4 marks)*

(b) Find a vector equation of the line AC. *(2 marks)*

(c) The point D has coordinates $(6, -1, p)$. It is given that the lines AC and BD intersect.

(i) Find the value of p. *(4 marks)*

(ii) Show that $ABCD$ is a rhombus, and state the length of each of its sides. *(4 marks)*

6 The points A, B and C have coordinates $(3, -2, 4)$, $(1, -5, 6)$ and $(-4, 5, -1)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 7 \\ -7 \\ 5 \end{bmatrix}$.

(a) Show that the point C lies on the line l. *(2 marks)*

(b) Find a vector equation of the line that passes through points A and B. *(3 marks)*

(c) The point D lies on the line through A and B such that the angle CDA is a right angle.

Find the coordinates of D. *(5 marks)*

(d) The point E lies on the line through A and B such that the area of triangle ACE is three times the area of triangle ACD.

Find the coordinates of the two possible positions of E. *(4 marks)*