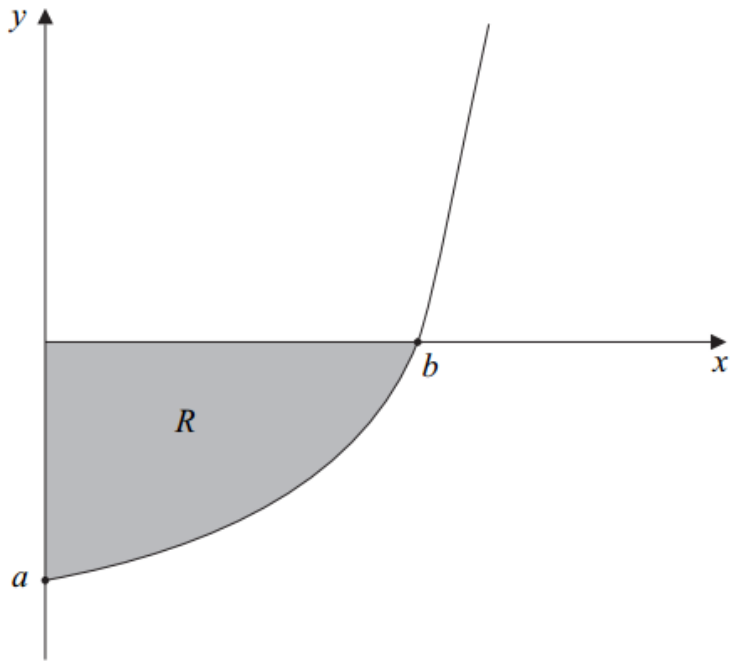
Core 3: Volumes of Revolution

Past Paper Questions
2006 - 2013

Name:

January 2006

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

June 2006

7 (a) Given that $z = \dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{dz}{dx} = \sec^2 x$. *(3 marks)*

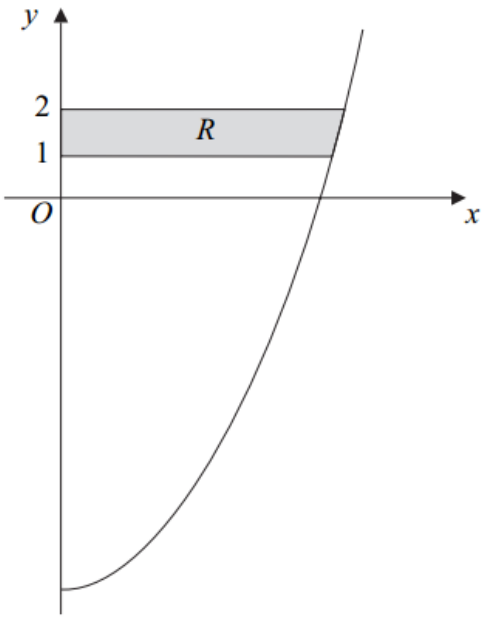
(b) Sketch the curve with equation $y = \sec x$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. *(2 marks)*

(c) The region R is bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(3 marks)*

January 2007

4 (c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



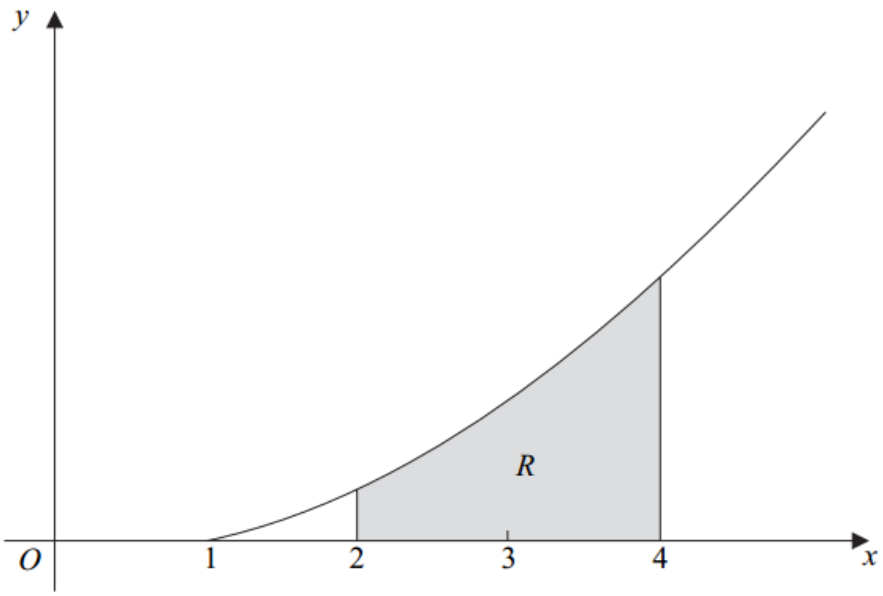
The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**. *(4 marks)*

June 2007

2 (a) Differentiate $(x - 1)^4$ with respect to x. *(1 mark)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

January 2008

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int xe^{-2x}\,dx$. *(4 marks)*

(c) A curve has equation $y = e^{-2x} + 6x$.

(i) Find the exact values of the coordinates of the stationary point of the curve. *(4 marks)*

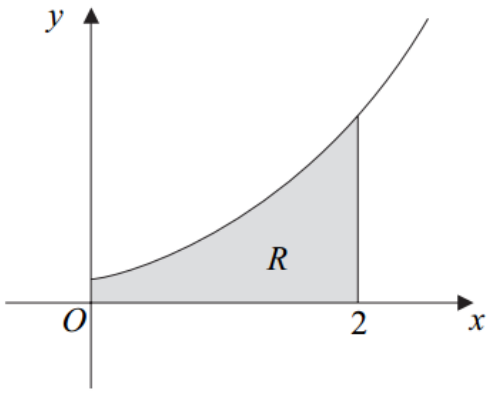
(ii) Determine the nature of the stationary point. *(2 marks)*

(iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(5 marks)*

June 2008

6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geqslant 0$.

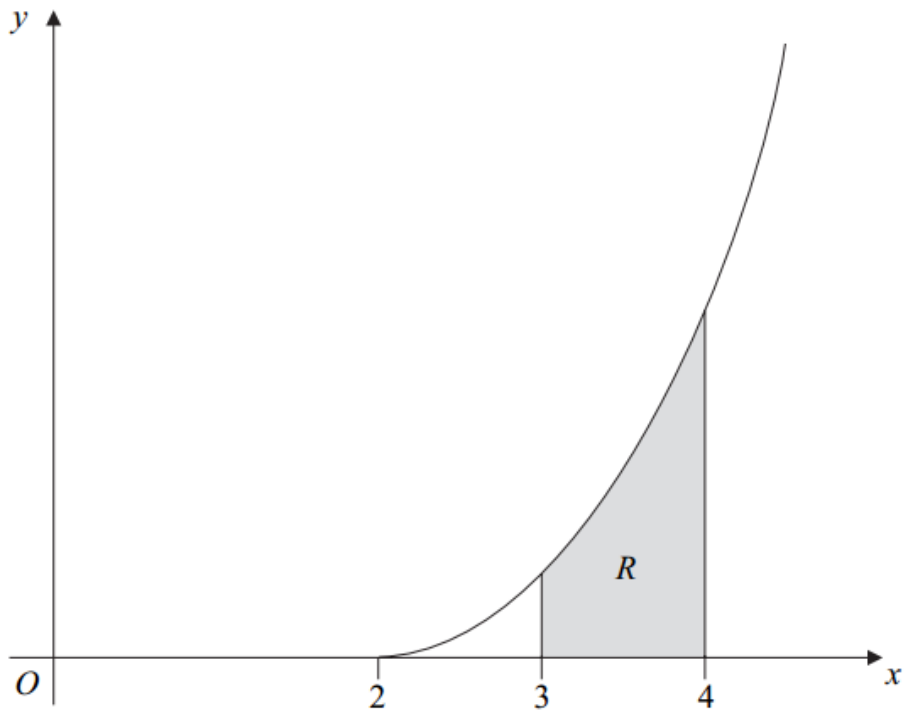


(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x-axis. *(4 marks)*

January 2009

2 The diagram shows the curve with equation $y = \sqrt{(x-2)^5}$ for $x \geqslant 2$.

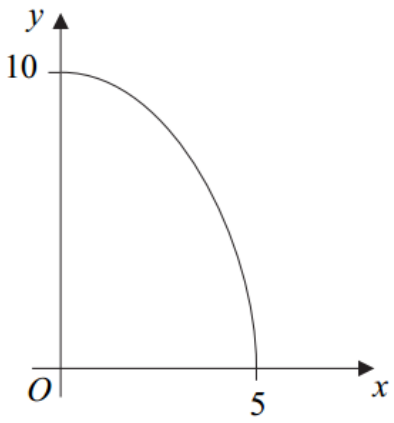


The shaded region R is bounded by the curve $y = \sqrt{(x-2)^5}$, the x-axis and the lines $x = 3$ and $x = 4$.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

June 2009

6 The diagram shows the curve with equation $y = \sqrt{100 - 4x^2}$, where $x \geqslant 0$.



(a) Calculate the volume of the solid generated when the region bounded by the curve shown above and the coordinate axes is rotated through 360° about the **y-axis**, giving your answer in terms of π. *(5 marks)*

January 2010

5 (b) A curve has equation $y = \ln(x^2 + 5)$.

(i) Show that this equation can be rewritten as $x^2 = e^y - 5$. *(1 mark)*

(ii) The region bounded by the curve, the lines $y = 5$ and $y = 10$ and the y-axis is rotated through 360° about the y-axis. Find the exact value of the volume of the solid generated. *(4 marks)*

June 2010

7 (a) Use integration by parts to find:

(i) $\int x \cos 4x \, dx$; *(4 marks)*

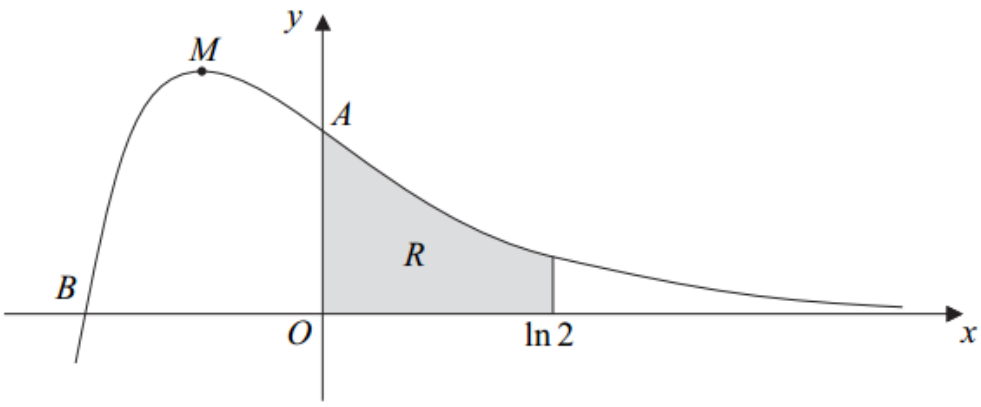
(ii) $\int x^2 \sin 4x \, dx$. *(4 marks)*

(b) The region bounded by the curve $y = 8x\sqrt{(\sin 4x)}$ and the lines $x = 0$ and $x = 0.2$ is rotated through 2π radians about the x-axis. Find the value of the volume of the solid generated, giving your answer to three significant figures. *(3 marks)*

January 2011

8 (a) Given that $e^{-2x} = 4$, find the exact value of x. *(2 marks)*

(b) The diagram shows the curve $y = 4e^{-2x} - e^{-4x}$.



The curve crosses the y-axis at the point A, the x-axis at the point B, and has a stationary point at M.

(iv) The shaded region R is bounded by the curve $y = 4e^{-2x} - e^{-4x}$, the lines $x = 0$ and $x = \ln 2$ and the x-axis.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\frac{p}{q}\pi$, where p and q are integers.

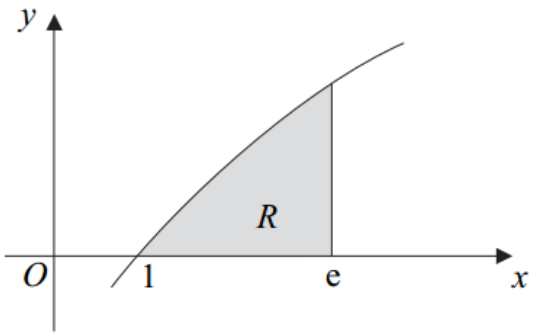
(7 marks)

June 2011

9 (a) Use integration by parts to find $\int x \ln x \, dx$. *(3 marks)*

(b) Given that $y = (\ln x)^2$, find $\frac{dy}{dx}$. *(2 marks)*

(c) The diagram shows part of the curve with equation $y = \sqrt{x} \ln x$.



The shaded region R is bounded by the curve $y = \sqrt{x} \ln x$, the line $x = e$ and the x-axis from $x = 1$ to $x = e$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in an exact form. *(6 marks)*

January 2012

7 (a) A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates. *(7 marks)*

(b) (i) Use integration by parts twice to find the exact value of $\int_0^4 x^2 e^{-\frac{x}{4}} \, dx$. *(7 marks)*

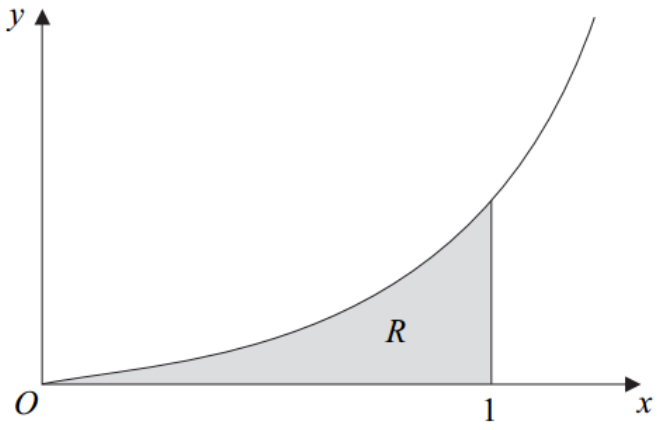
(ii) The region bounded by the curve $y = 3xe^{-\frac{x}{8}}$, the x-axis from 0 to 4 and the line $x = 4$ is rotated through 360° about the x-axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated. *(2 marks)*

June 2012

4 (a) By using integration by parts, find $\int x\,e^{6x}\,dx$. *(4 marks)*

(b) The diagram shows part of the curve with equation $y = \sqrt{x}\,e^{3x}$.

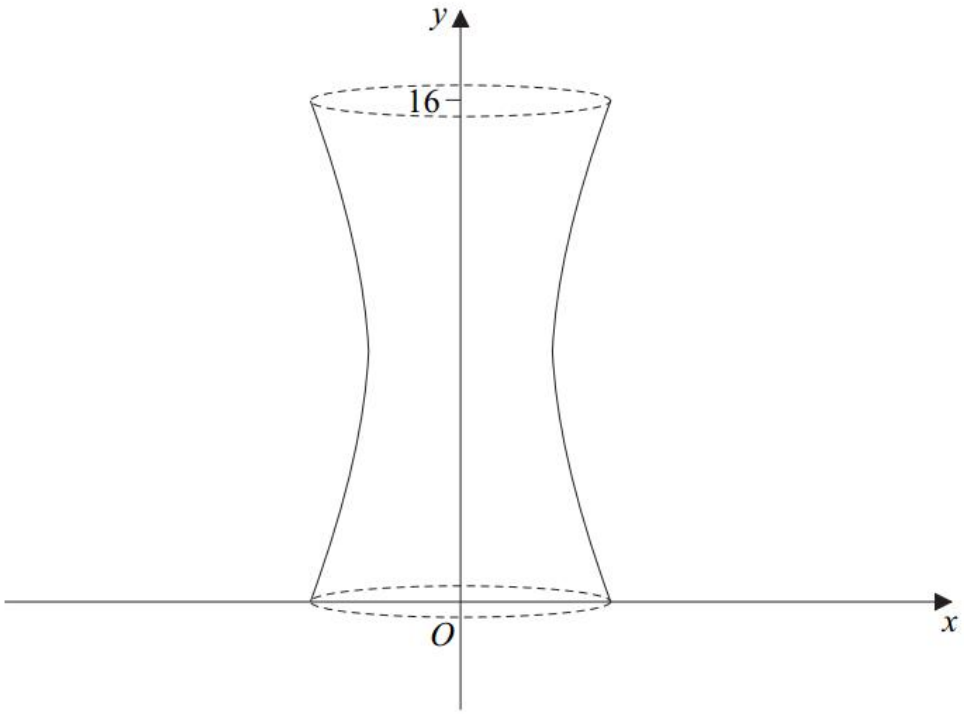


The shaded region R is bounded by the curve $y = \sqrt{x}\,e^{3x}$, the line $x = 1$ and the x-axis from $x = 0$ to $x = 1$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\pi(pe^6 + q)$, where p and q are rational numbers. *(3 marks)*

June 2013

9 The shape of a vase can be modelled by rotating the curve with equation $16x^2 - (y - 8)^2 = 32$ between $y = 0$ and $y = 16$ completely **about the *y*-axis**.



The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value. *(5 marks)*