Core 2: Sequences and Series

Past Paper Questions
2006-2013

Name:

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\left[2a + (n-1)d\right]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

January 2006

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

June 2006

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

January 2007

5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

June 2007

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

January 2008

2 The arithmetic series

$$51 + 58 + 65 + 72 + \ldots + 1444$$

has 200 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 101st term of the series. *(2 marks)*

(c) Find the sum of **the last** 100 terms of the series. *(2 marks)*

June 2008

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \ldots$$

(a) Find the common ratio of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

January 2009

8 The 25th term of an arithmetic series is 38.

The sum of the first 40 terms of the series is 1250.

(a) Show that the common difference of this series is 1.5. *(6 marks)*

(b) Find the number of terms in the series which are less than 100. *(3 marks)*

June 2009

3 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = ku_n + 12$$

where k is a constant.

The first two terms of the sequence are given by

$$u_1 = 16 \qquad u_2 = 24$$

(a) Show that $k = 0.75$. *(2 marks)*

(b) Find the value of u_3 and the value of u_4. *(2 marks)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

7 A geometric series has second term 375 and fifth term 81.

(a) (i) Show that the common ratio of the series is 0.6. *(3 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) The nth term of the series is u_n. Find the value of $\sum_{n=6}^{\infty} u_n$. *(4 marks)*

January 2010

4 An arithmetic series has first term a and common difference d.

The sum of the first 31 terms of the series is 310.

(a) Show that $a + 15d = 10$. *(3 marks)*

(b) Given also that the 21st term is twice the 16th term, find the value of d. *(3 marks)*

(c) The nth term of the series is u_n. Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(4 marks)*

June 2010

2 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = 6 + \frac{2}{5}u_n$$

The first term of the sequence is given by $u_1 = 2$.

(a) Find the value of u_2 and the value of u_3. *(2 marks)*

(b) The limit of u_n as n tends to infinity is L.

Write down an equation for L and hence find the value of L. *(3 marks)*

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5 (a) An infinite geometric series has common ratio r.

The first term of the series is 10 and its sum to infinity is 50.

(i) Show that $r = \frac{4}{5}$. *(2 marks)*

(ii) Find the second term of the series. *(2 marks)*

(b) The first and second terms of the geometric series in part **(a)** have the same values as the 4th and 8th terms respectively of an arithmetic series.

(i) Find the common difference of the arithmetic series. *(3 marks)*

(ii) The nth term of the arithmetic series is u_n. Find the value of $\sum_{n=1}^{40} u_n$. *(4 marks)*

January 2011

6 A geometric series has third term 36 and sixth term 972.

(a) (i) Show that the common ratio of the series is 3. *(2 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) The nth term of the series is u_n.

(i) Show that $\sum_{n=1}^{20} u_n = 2(3^{20} - 1)$. *(2 marks)*

(ii) Find the least value of n such that $u_n > 4 \times 10^{15}$. *(3 marks)*

June 2011

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

January 2012

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

June 2012

1 The arithmetic series

$$23 + 32 + 41 + 50 + \ldots + 2534$$

has 280 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 100th term of the series. *(2 marks)*

(c) Find the sum of the 280 terms of the series. *(2 marks)*

4 The nth term of a geometric series is u_n, where $u_n = 48\left(\frac{1}{4}\right)^n$.

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Find the value of the common ratio of the series. *(1 mark)*

(c) Find the sum to infinity of the series. *(2 marks)*

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. *(3 marks)*

January 2013

6 (a) A geometric series begins $420 + 294 + 205.8 + \ldots$.

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*