

C3 Functions Challenge

Challenge 1

The function f is defined by

$$f: x \rightarrow \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \frac{2}{x-1}$, $x > 1$. (4)

(b) Find $f^{-1}(x)$. (3)

The function g is defined by

$$g: x \rightarrow x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$. (3)



Challenge 2

Figure 1

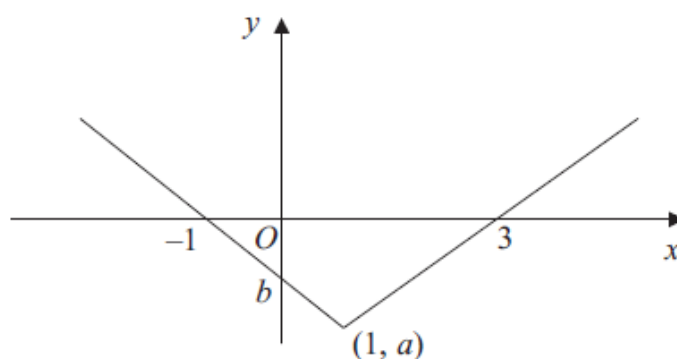


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)



Challenge 3

Figure 1

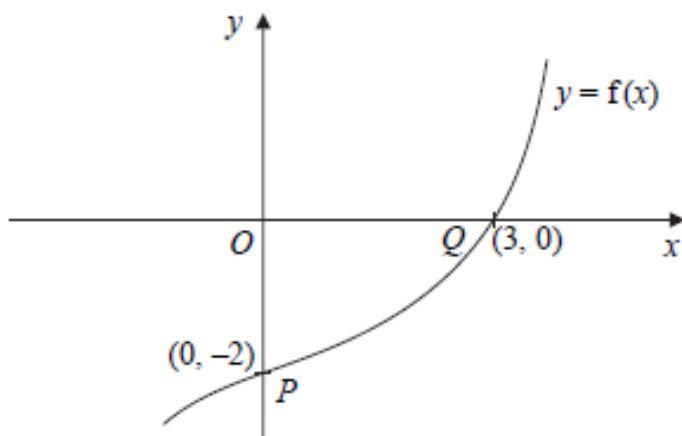


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2}f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



Final Challenge

For the constant k , where $k > 1$, the functions f and g are defined by

$$f: x \mapsto \ln(x + k), \quad x > -k,$$

$$g: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f .

(1)

(c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k .

(4)

