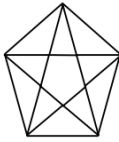
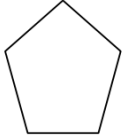


A graph  $G$  has five vertices.

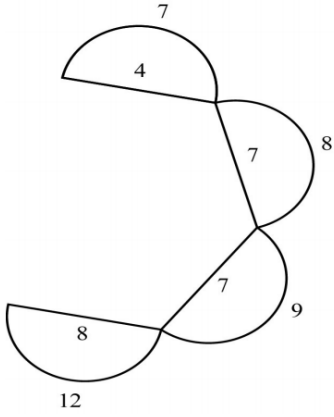
- (a) (i) Given that  $G$  is connected, state the number of arcs on a minimum spanning tree. *(1 mark)*
- (ii) Given that  $G$  is Eulerian with all vertices having the same degree  $d$ , state the minimum value of  $d$ . *(1 mark)*
- (iii) Given that  $G$  has a Hamiltonian cycle, state the number of edges in such a cycle. *(1 mark)*
- (b) Given that  $G$  has all the properties mentioned in part (a), draw a possible graph  $G$ . *(1 mark)*

<b>2(a)(i)</b>	4		B1	1	or equivalent or 
<b>(ii)</b>	2		B1	1	
<b>(iii)</b>	5		B1	1	
<b>(b)</b>			B1	1	
<b>Total</b>				<b>4</b>	

A connected graph has five vertices and has arc lengths of

4, 7, 7, 7, 8, 8, 9 and 12 units.

- (a) State the minimum length of a minimum spanning tree for any such graph. *(1 mark)*
- (b) State the minimum length of a Hamiltonian cycle for any such graph. *(1 mark)*
- (c) State the minimum length of an Eulerian cycle for any such graph. *(1 mark)*
- (d) In the case when the length of its minimum spanning tree is 26 units, draw a sketch to show a possible graph. *(3 marks)*

Q	Solution	Marks	Total	Comments
5 (a)	$\text{Min} = 4 + 7 + 7 + 7$ $= 25$	B1	1	
(b)	$\text{Min (H)} = 4 + 7 + 7 + 7 + 8$ $= 33$	B1	1	
(c)	$\text{Min (E)} = \Sigma = 62$	B1	1	
(d)		M1 m1 A1	3	5 vertices 8 edges All correct
		<b>Total</b>	<b>6</b>	

(a) A connected graph has four vertices. State the number of edges in the graph's minimum spanning tree. (1 mark)

(b) A graph has  $n$  vertices. The graph is complete, i.e. each vertex is joined to every other vertex by exactly one edge.

(i) State the number of edges in the graph's minimum spanning tree. (1 mark)

(ii) Determine the number of Hamiltonian cycles in the graph. (2 marks)

(c) A connected graph has four vertices and has arc lengths of

4, 4.5, 5, 6.5, 7, 8 and 9 units.

The length of its minimum spanning tree is 17 units. Draw a sketch to show a possible graph. (3 marks)

Q	Solution	Marks	Total	Comments
4 (a)	3	B1	1	
(b)(i)	$n-1$	B1	1	
(ii)	$(n-1)!$	M1A1	2	M1 for factorial
(c)		M1 A1 A1	3	connected graph m.s.t. = 17 all other arcs used and correct
<b>Total</b>			<b>7</b>	

A graph has eight vertices labelled 1, 2, 3, 4, 5, 6, 7, 8. Two vertices are joined by an edge if their numbers differ by one or two. For example, 2 and 3 are joined, 4 and 6 are joined, but 5 and 8 are not joined.

- (a) Draw a picture of the graph. (3 marks)
- (b) Explain how you know that the graph is semi-Eulerian. Give an example of an Eulerian trail in the graph. (3 marks)
- (c) Give an example of a Hamiltonian cycle in the graph. (2 marks)
- (d) List all paths of length four from vertex 1 to vertex 8. (3 marks)

3 (a)		M1 A1 A1	3	△ two rest
(b)	Two odd vertices, 2 and 7 e.g. 2312 4354657687	B1 M1 A1	3	
(c)	e.g. 124687531	M1A1	2	
(d)	13578; 13568; 13468; 12468	M1 A1 A1	3	for two rest
<b>Total</b>			<b>11</b>	