D1 Graphs and Networks

Challenge 1

A graph G has five vertices.

(a)   (i)   Given that G is connected, state the number of arcs on a minimum spanning tree.

(1 mark)

        (ii)   Given that G is Eulerian with all vertices having the same degree d, state the minimum value of d.

(1 mark)

        (iii)   Given that G has a Hamiltonian cycle, state the number of edges in such a cycle.

(1 mark)

(b)   Given that G has all the properties mentioned in part (a), draw a possible graph G.   *(1 mark)*

Challenge 2

A connected graph has five vertices and has arc lengths of

$$4, \ 7, \ 7, \ 7, \ 8, \ 8, \ 9 \ \text{and} \ 12 \ \text{units.}$$

(a) State the minimum length of a minimum spanning tree for any such graph.   *(1 mark)*

(b) State the minimum length of a Hamiltonian cycle for any such graph.   *(1 mark)*

(c) State the minimum length of an Eulerian cycle for any such graph.   *(1 mark)*

(d) In the case when the length of its minimum spanning tree is 26 units, draw a sketch to show a possible graph.   *(3 marks)*

Challenge 3

(a) A connected graph has four vertices. State the number of edges in the graph's minimum spanning tree.   *(1 mark)*

(b) A graph has n vertices. The graph is complete, i.e. each vertex is joined to every other vertex by exactly one edge.

   (i) State the number of edges in the graph's minimum spanning tree.   *(1 mark)*

   (ii) Determine the number of Hamiltonian cycles in the graph.   *(2 marks)*

(c) A connected graph has four vertices and has arc lengths of

$$4, \ 4.5, \ 5, \ 6.5, \ 7, \ 8 \ \text{and 9 units.}$$

The length of its minimum spanning tree is 17 units. Draw a sketch to show a possible graph.
                                                                                  (3 marks)

Final Challenge

A graph has eight vertices labelled 1, 2, 3, 4, 5, 6, 7, 8. Two vertices are joined by an edge if their numbers differ by one or two. For example, 2 and 3 are joined, 4 and 6 are joined, but 5 and 8 are not joined.

(a) Draw a picture of the graph.   *(3 marks)*

(b) Explain how you know that the graph is semi-Eulerian. Give an example of an Eulerian trail in the graph.   *(3 marks)*

(c) Give an example of a Hamiltonian cycle in the graph.   *(2 marks)*

(d) List all paths of length four from vertex 1 to vertex 8.   *(3 marks)*