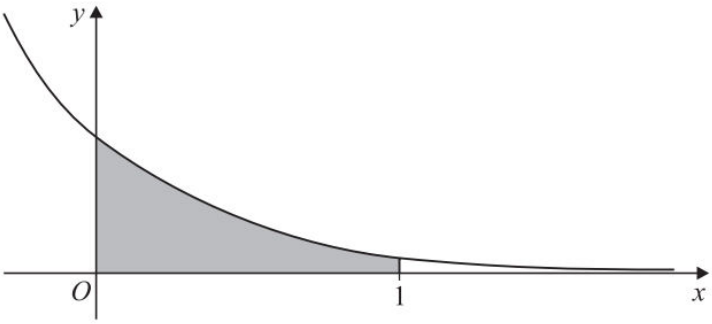
Use integration by parts to find

$$\int x^2 e^{-x} \, dx.$$ **(7)**

2.	$u = x^2, \; u' = 2x, \; v' = e^{-x}, \; v = -e^{-x}$	M1 A1	
	$I = -x^2 e^{-x} - \int -2x e^{-x} \, dx = -x^2 e^{-x} + \int 2x e^{-x} \, dx$	A2	
	$u = 2x, \; u' = 2, \; v' = e^{-x}, \; v = -e^{-x}$	M1	
	$I = -x^2 e^{-x} - 2x e^{-x} - \int -2e^{-x} \, dx$	A1	
	$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c$	A1	(7)



The diagram shows the graph of

$$y = e^{-2x}.$$

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. *(3 marks)*

(b) (i) Find $\int y \, dx$. *(2 marks)*

(ii) Hence show that the area of the region shaded on the diagram is

$$\frac{e^2 - 1}{2e^2}.$$ *(3 marks)*

Q	Solution	Marks	Total	Comments
4 (a)	$y' = pe^{-2x}$	M1		Where p is a constant, $p = \pm 2$ or $\pm\frac{1}{2}$ or ± 1
	$p = -2$	A1		
	$y'' = 4e^{-2x}$	A1F	3	ft consistent errors provided $p \neq 1$
(b)(i)	$\int y \, dx = qe^{-2x} (+c)$	M1		Where q is a constant, $q = \pm 2$ or $\pm\frac{1}{2}$ or ± 1
	$q = -\frac{1}{2}$	A1F	2	ft wrong value of p provided $p \neq 1$
(ii)	Area $= \int_0^1 y \, dx$	M1		Allow even if formula not used; condone $\int_1^0 y \, dx$
	$\ldots = -\frac{1}{2}e^{-2} + \frac{1}{2}$	A1F		$e^0 = 1$ must be used; ft wrong coefficient in (i)
	$\ldots = \frac{e^2 - 1}{2e^2}$	A1	3	convincingly obtained (AG)
	Total		**8**	

Use the substitution $x = 2\tan u$ to show that

$$\int_0^2 \frac{x^2}{x^2+4}\,dx = \tfrac{1}{2}(4-\pi).$$ **(8)**

2. $x = 2\tan u \Rightarrow \frac{dx}{du} = 2\sec^2 u$ M1

$x = 0 \Rightarrow u = 0,\ x = 2 \Rightarrow u = \frac{\pi}{4}$ B1

$I = \int_0^{\frac{\pi}{4}} \frac{4\tan^2 u}{4\sec^2 u} \times 2\sec^2 u\ du = \int_0^{\frac{\pi}{4}} 2\tan^2 u\ du$ A1

$= \int_0^{\frac{\pi}{4}} (2\sec^2 u - 2)\ du$ M1

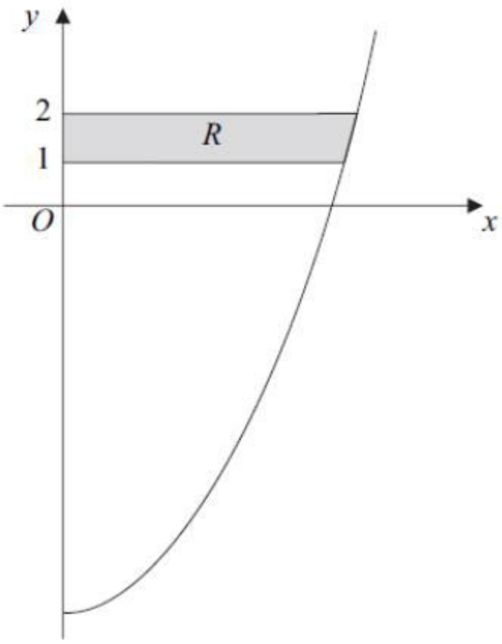
$= [2\tan u - 2u]_0^{\frac{\pi}{4}}$ M1 A1

$= (2 - \frac{\pi}{2}) - (0) = \frac{1}{2}(4-\pi)$ M1 A1 **(8)**

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2+5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**. *(4 marks)*

4(a)	$\int x\sin x\,dx \quad u=x$			
	$\frac{dv}{dx}=\sin x$	M1		For differentiating one term and integrating other
	$\frac{du}{dx}=1 \quad v=-\cos x$			
	$\int = -x\cos x-\int -\cos x(dx)$	m1		For correctly substituting their terms into parts formula
		A1		
	$=-x\cos x+\sin x\ (+c)$	A1	4	CSO
(b)	$u=x^2+5$			
	$du=2x\,dx$			
	$\int = \int \frac{1}{2}u^{\frac{1}{2}}(du)$	M1		$\int ku^{\frac{1}{2}}(du)$ condone omission of du but M0 if dx
		A1		$k=\frac{1}{2}$ OE
	$=\frac{u^{\frac{3}{2}}}{3}$	A1√		Ft $\int ku^{\frac{1}{2}}du$
	$=\frac{1}{3}\sqrt{(x^2+5)^3}\ (+c)$	A1	4	CSO **SC** $\frac{2}{6}\sqrt{(x^2+5)^3}$ with no working B3
(c)	$y=x^2-9$			
	$x^2=y+9$			
	$V=\pi\int x^2 dy$	B1		Must have π and x^2, condone omission of dy, but B0 if dx
	$=\pi\int(y+9)dy$			
	$=(\pi)\left[\frac{y^2}{2}+9y\right]_1^2$ or $(\pi)\left[\frac{(y+9)^2}{2}\right]_1^2$	M1		$\int$"their x^2"dy integrated; Limits 2 and 1 substituted in correct order including – sign (π not necessary)
	$=(\pi)\left[20-9\frac{1}{2}\right]$	m1		
	$=10\frac{1}{2}\pi$	A1	4	CSO
	Total		**12**	