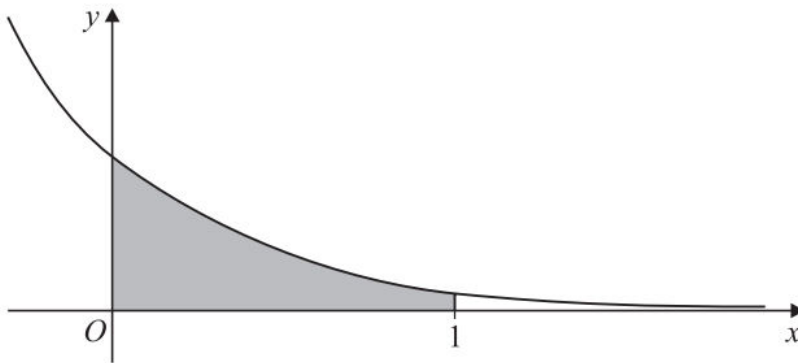# C3 Integration Challenge

Challenge 1

Use integration by parts to find

$$\int x^2 e^{-x} \, dx. \tag{7}$$

Challenge 2

The diagram shows the graph of

$$y = e^{-2x}.$$

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.   *(3 marks)*

(b) (i) Find $\int y\, dx$.   *(2 marks)*

  (ii) Hence show that the area of the region shaded on the diagram is

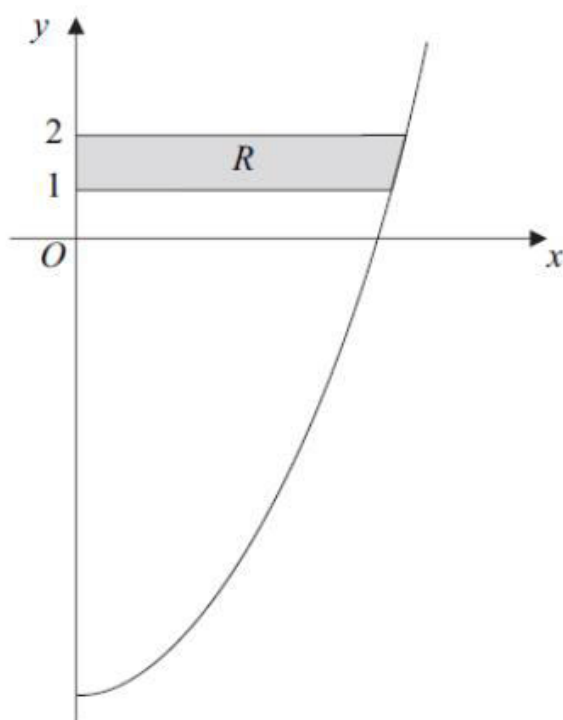$$\frac{e^2 - 1}{2e^2}.$$ *(3 marks)*

Challenge 3

Use the substitution $x = 2\tan u$ to show that

$$\int_0^2 \frac{x^2}{x^2+4}\ \mathrm{d}x = \tfrac{1}{2}(4-\pi).$$

(8)

Final Challenge

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x \sqrt{x^2 + 5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.

The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through $360°$ **about the y-axis**. *(4 marks)*