6 A particle P moves so that at time t seconds its velocity $\mathbf{v}$ m s^{-1} is

$$\mathbf{v} = 2t\,\mathbf{i} + 4\mathbf{j}, \qquad t \geqslant 0.$$

(a) At time $t = 0$, the particle is at the point with position vector $2\mathbf{j}$ metres.

Find the position vector of P at time t. *(4 marks)*

(b) At times $t = 2$ and $t = 4$, the particle passes through the points C and D respectively.

Find the vector $\overrightarrow{CD}$. *(4 marks)*

6	**(a)**	$\mathbf{r} = \int (2t\mathbf{i} + 4\mathbf{j})\,dt$	M1		Integration attempted for M1
		$= t^2\mathbf{i} + 4t\mathbf{j} (+\mathbf{c})$	A1		
		$t = 0,\ \mathbf{r} = 2\mathbf{j},\ 2\mathbf{j} = \mathbf{c}$	m1		
		$\mathbf{r} = t^2\mathbf{i} + (4t + 2)\mathbf{j}$	A1F	4	Any vector form
	(b)	$t = 2,\ \mathbf{r} = 4\mathbf{i} + 10\mathbf{j}$	M1 A1F		subs either value of t into $\mathbf{r}$ for M1
		$t = 4,\ \mathbf{r} = 16\mathbf{i} + 18\mathbf{j}$	A1F		ft for vectors with 2 non zero components
		$\overrightarrow{CD} = 12\mathbf{i} + 8\mathbf{j}$	A1F	4	subtraction of two vectors
					Alternative
					$\left[t^2\mathbf{i} + 4t\mathbf{j}\right]_2^4$ M1
					$= (16\mathbf{i} + 16\mathbf{j}) - (4\mathbf{i} + 8\mathbf{j})$ A1F A1F
					$= 12\mathbf{i} + 8\mathbf{j}$ A1F
					subtraction
		Total		**8**	

1 A particle moves so that at time, t seconds, its position, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = \left(t^3 - 3t^2\right)\mathbf{i} + \left(4t + 2t^2\right)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) By differentiating, find the velocity of the particle at time t. *(2 marks)*

(b) Find, but do not simplify, an expression for the magnitude of the acceleration of the particle. *(4 marks)*

(c) Find the time when the magnitude of the acceleration is a minimum and find its magnitude at this time. *(3 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
1 (a)	$\mathbf{v} = \left(3t^2 - 6t\right)\mathbf{i} + (4 + 4t)\mathbf{j}$	M1 A1	2	Differentiating both components Correct answer
(b)	$\mathbf{a} = (6t - 6)\,\mathbf{i} + 4\mathbf{j}$	M1 A1		Differentiating the velocity Correct acceleration
	$a = \sqrt{(6t-6)^2 + 4^2}$	m1 A1	4	Finding magnitude (must include square root) Correct expression
(c)	$6t - 6 = 0$ $t = 1$	M1 A1		**i** component equal to zero $t = 1$ **or** M1: Differentiating A1: $t = 1$
	$a = 4$	B1	3	$a = 4$ **not** $4\mathbf{j}$
	Total		**9**	

5 A sky diver jumps at time $t = 0$ from an aeroplane that is travelling horizontally. The velocity, $\mathbf{v}$ m s^{-1}, of the sky diver at time t seconds is given by

$$\mathbf{v} = 70e^{-0.1t}\,\mathbf{i} + 40(e^{-0.1t} - 1)\,\mathbf{j}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the horizontal and upward vertical directions respectively.

(a) Describe what happens to the velocity of the sky diver as t increases. *(2 marks)*

(b) Taking the origin to be the initial position of the sky diver, find an expression for his position vector at time t seconds. *(6 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
5(a)	Vertical component increases towards 40 m s^{-1}	B1		
	Horizontal component decreases to zero	B1	2	
(b)	$\mathbf{r} = \int 70e^{-0.1t}dt\mathbf{i} + \int 40e^{-0.1t} - 40dt\mathbf{j}$	M1		Integrates $\mathbf{v}$
		A1		For each component, ignore
	$= \left(-700e^{-0.1t} + c\right)\mathbf{i} + (-400e^{-0.1t} - 40t + d)\mathbf{j}$	A1		constants
	Initial conditions imply	M1		Finds constants
	$c = 700, d = 400$	A1		
	$\mathbf{r} = \left(700 - 700e^{-0.1t}\right)\mathbf{i} + (400 - 400e^{-0.1t} - 40t)\mathbf{j}$	A1	6	For each constant
	Total		**8**	

7 A boat moves so that its position vector, **r** metres, at time t seconds is

$$\mathbf{r} = 40\cos\left(\frac{t}{20}\right)\mathbf{i} + 80\sin\left(\frac{t}{20}\right)\mathbf{j}$$

The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the velocity of the boat at time t. *(3 marks)*

(b) In what direction is the boat travelling when $t = 0$? Justify your answer. *(2 marks)*

(c) At what time is the boat travelling due south for the first time? *(2 marks)*

Question Number and part	Solution	Marks	Total	Comments
7(a)	$\mathbf{v} = -2\sin\left(\frac{t}{20}\right)\mathbf{i} + 4\cos\left(\frac{t}{20}\right)\mathbf{j}$	M1 A1 A1	3	Differentiating **i** component correct **j** component correct
(b)	$\mathbf{v} = 4\mathbf{j}$ Travelling north	B1 B1	2	Correct velocity at $t = 0$ Travelling north with $\mathbf{v} = n\mathbf{j}$ where $n > 0$
(c)	$\frac{t}{20} = \pi$	M1		Equation for t based on $0\mathbf{i}$
	$t = 20\pi$	A1	2	Correct solution
	Total		**7**	