M2 Kinematics Challenge

Challenge 1

A particle P moves so that at time t seconds its velocity $\mathbf{v}$ m s^{-1} is

$$\mathbf{v} = 2t\,\mathbf{i} + 4\mathbf{j}, \qquad t \geqslant 0.$$

(a) At time $t = 0$, the particle is at the point with position vector $2\mathbf{j}$ metres.

Find the position vector of P at time t. *(4 marks)*

(b) At times $t = 2$ and $t = 4$, the particle passes through the points C and D respectively.

Find the vector $\overrightarrow{CD}$. *(4 marks)*

A particle moves so that at time, t seconds, its position, $\mathbf{r}$ metres, is given by

$$\mathbf{r} = \left(t^3 - 3t^2\right)\mathbf{i} + \left(4t + 2t^2\right)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) By differentiating, find the velocity of the particle at time t. *(2 marks)*

(b) Find, but do not simplify, an expression for the magnitude of the acceleration of the particle. *(4 marks)*

(c) Find the time when the magnitude of the acceleration is a minimum and find its magnitude at this time. *(3 marks)*

Challenge 3

A sky diver jumps at time $t = 0$ from an aeroplane that is travelling horizontally. The velocity, v m s^{-1}, of the sky diver at time t seconds is given by

$$\mathbf{v} = 70e^{-0.1t}\,\mathbf{i} + 40(e^{-0.1t} - 1)\,\mathbf{j}$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the horizontal and upward vertical directions respectively.

(a) Describe what happens to the velocity of the sky diver as t increases. *(2 marks)*

(b) Taking the origin to be the initial position of the sky diver, find an expression for his position vector at time t seconds. *(6 marks)*

Final Challenge

A boat moves so that its position vector, **r** metres, at time t seconds is

$$\mathbf{r} = 40\cos\left(\frac{t}{20}\right)\mathbf{i} + 80\sin\left(\frac{t}{20}\right)\mathbf{j}$$

The unit vectors **i** and **j** are directed east and north respectively.

(a) Find an expression for the velocity of the boat at time t.   *(3 marks)*

(b) In what direction is the boat travelling when $t = 0$? Justify your answer.   *(2 marks)*

(c) At what time is the boat travelling due south for the first time?   *(2 marks)*