Kinematics Badge Challenge

Challenge 1

A particle moves in the horizontal plane that contains the perpendicular unit vectors **i** and **j**. Initially it is at the origin and has velocity $18\mathbf{i}\ \text{ms}^{-1}$. After accelerating for 10 seconds its velocity is $(30\mathbf{i} + 8\mathbf{j})\ \text{ms}^{-1}$. Assume that the acceleration of the particle is constant.

(a) Find the acceleration of the particle. *(2 marks)*

(b) Find the position vector of the particle when its velocity is $(36\mathbf{i} + 12\mathbf{j})\ \text{ms}^{-1}$. *(6 marks)*

Challenge 2

A car, of mass 900 kg, is initially at rest. On a short journey the car

I. accelerates uniformly for T seconds to a speed of $20\ \text{ms}^{-1}$,
II. then travels at this speed for a period of time,
III. then decelerates uniformly for $2T$ seconds before coming to rest.

(a) In one journey the car moves for a total of 40 seconds and travels a total of 620 m. Using this information:

(i) sketch a velocity-time graph and hence, or otherwise, find T; *(5 marks)*

(ii) calculate the magnitude of the resultant force on the car, during each stage of the journey; *(2 marks)*

(iii) sketch a graph to show how the resultant force acting on the car varies with time; *(3 marks)*

(iv) find the speed of the car after it has travelled 20 m. *(3 marks)*

(b) In the case when $T = 5$, find the time that it would take the car to complete a 1000 m journey. *(3 marks)*

Challenge 3

At time $t = 0$, a boat is travelling due east at a speed of $3\,\text{m s}^{-1}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Write down the initial velocity of the boat in vector form. *(1 mark)*

(b) The boat has a constant acceleration of $(0.1\mathbf{i} + 0.2\mathbf{j})\,\text{m s}^{-2}$. Find an expression for the velocity of the boat at time t seconds. *(2 marks)*

(c) When $t = T$, the boat is travelling north east. Form an equation that T must satisfy, and solve it to show that $T = 30$. *(4 marks)*

(d) Find the distance of the boat from its initial position when $t = 20$. *(5 marks)*

Final Challenge

At time $t = 0$, a particle is at the origin and moving with velocity $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. When $t = 10$ seconds the position vector of the particle is $(44\mathbf{i} + 28\mathbf{j})$ metres. The particle is moving with constant acceleration.

(a) Find the acceleration of the particle. *(4 marks)*

(b) Find the position vector of the particle at time t. *(3 marks)*

(c) At time $t = T$, the position vector of the particle is $(96\mathbf{i} + 72\mathbf{j})$ metres.

(i) Find T. *(4 marks)*

(ii) Find the speed of the particle at this time. *(3 marks)*