Linear Graphs

The points A and B have coordinates (13, 5) and (9, 2) respectively.

(a) (i) Find the gradient of AB. *(1 mark)*

(ii) Find an equation for the line AB. *(1 mark)*

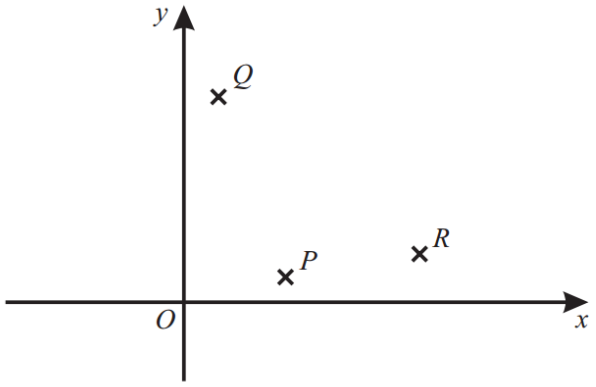
(b) The point C has coordinates (2, 3) and the point X lies on AB so that XC is perpendicular to AB.

(i) Show that the equation of the line XC can be written in the form $4x + 3y = 17$. *(4 marks)*

(ii) Calculate the coordinates of X. *(3 marks)*

4(a)(i)	Gradient $AB = \frac{3}{4}$	B1	1	
(ii)	$y - 2 = \frac{3}{4}(x - 9)$	B1	1	$y = \frac{3}{4}x - \frac{19}{4}$ oe
(b)(i)	Gradient $XC = \frac{-1}{\text{grad } AB}$	M1		Or awareness that $m_1 m_2 = -1$
	$= -\frac{4}{3}$	A1√		
	$y - 3 = -\frac{4}{3}(x - 2)$	M1		ft their gradient
	$3y - 9 = -4x + 8 \Rightarrow 4x + 3y = 17$	A1	4	ag
(ii)	Solving 'their' AB with $4x + 3y = 17$	M1		Realising the need to use these equations
	$x = \ldots$ or $y = \ldots$	m1		Elimination/substitution to solve for x/y
	$x = 5, \quad y = -1$	A1	3	$(5, -1)$ are coordinates
	Total		**9**	

The points P, Q and R have coordinates (3,1), (1,9) and (7,2) respectively.



(a) Find an equation for the straight line QR in the form $ax + by = c$, where a, b and c are integers. *(3 marks)*

(b) Prove that the triangle PQR is right-angled and find its area. *(4 marks)*

(c) Determine an equation for the straight line which passes through P and which is perpendicular to QR. *(2 marks)*

4 (a)	$y-9=-\frac{7}{6}(x-1)$ etc	M1 A1		good attempt at any form of line equation
	$\underline{7x+6y=61}$	A1 √	**(3)**	
(b)	grad $PQ=-4$; grad $PR=\frac{1}{4}$	M1		Attempt at both gradients (or Pythagoras etc)
	product of grads $=-4\times\frac{1}{4}=-1$	A1		3 sides for Pythagoras $PQ^2=68$; $PR^2=17$; $QR^2=85$
	Area $=\frac{1}{2}PQ\times PR$ and attempt to calculate PQ, PR	M1		
	$=\frac{1}{2}\left[\sqrt{68}\times\sqrt{17}\right]=\underline{17}$	A1	**(4)**	
(c)	grad $QR=-\frac{7}{6}$ $\Rightarrow$ grad of perp $=\frac{6}{7}$	M1		
	Equation of line is $(y-1)=\frac{6}{7}(x-3)$	A1 √	**(2)**	
		TOTAL	**9**	

The points A, B and C have coordinates (1,7), (5,5) and (7,9) respectively.

(a) Show that AB and BC are perpendicular. *(3 marks)*

(b) Find an equation of the line BC. *(2 marks)*

(c) The equation of the line AC is $3y = x + 20$ and M is the midpoint of AB.

(i) Find an equation of the line through M parallel to AC. *(3 marks)*

(ii) This line intersects BC at the point T. Find the coordinates of T. *(3 marks)*

2 (a)	Gradient $BC = \frac{9-5}{7-5}$; {=2}			
	Gradient $AB = \frac{5-7}{5-1}$; $\{=-0.5\}$	M1 A1		Attempt at both gradients Both correct
	$\{2 \times -0.5 = -1\} \Rightarrow AB$ and BC perp.	A1 cso	**(3)**	AG Completed convincingly
(b)	Eqn. of line BC $y - 5 = 2(x - 5)$ OE	M1 A1 ft	**(2)**	Accept any valid form
(c)(i)	Coords. $M(3,6)$ Line parallel to AC: $3y = x + C$. passes thro' (3,6) so $3y = x + 15$ (B)	B1 M1 A1 ft	**(3)**	$c \neq 20$ only ft if one coordinate of M is correct Accept any valid form
(ii)	Eqn. of line BC: $y = 2x - 5$ (A) Solve c's (A) and c's (B) simultaneously	M1		As far as a value for x or y
	$5y = 35 \Rightarrow y = 7$. **OR** $5x = 30 \Rightarrow x = 6$	A1		
	$\Rightarrow T(6, 7)$	A1	**(3)**	[Award 3 marks if correct answer with no working]
		TOTAL	**(11)**	

The line AB has equation $3x - 4y = 4$, and the line BC has equation $4x + 3y = 22$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Prove that the lines AB and BC are perpendicular. *(2 marks)*

(b) Find the coordinates of the point B. *(3 marks)*

(c) The point A has coordinates $(4p, 3p - 1)$, where p is a constant, and the point C has coordinates $(1, 6)$.

(i) Show that $AC^2 = 25p^2 - 50p + 50$. *(2 marks)*

(ii) Given that AC has length $\sqrt{125}$, find the possible values of p. *(3 marks)*

3(a)(i)	Attempt to rearrange to $y = ...$	M1		$y = \frac{3}{4}x - 1$
	$\Rightarrow$ gradient $AB = \frac{3}{4}$	A1	2	Condone correct answer for gradient even if equation is incorrect.
(ii)	gradient $BC = -\frac{4}{3}$			
	Attempt at $m_{AB} \times m_{BC} = \frac{3}{4} \times -\frac{4}{3}$	M1		
	$= -1 \Rightarrow AB$ and BC are perpendicular	A1	2	Or awareness that $m_1 \times m_2 = -1$
(b)	Attempt to eliminate x or y	M1		$3x - 4y = 4; \quad 4x + 3y = 22$
	$x = 4,$	A1		
	$y = 2$	A1	3	Coordinates are (4,2)
(c)(i)	$(4p-1)^2 + (3p-7)^2$	M1		Or Pythagoras $AB^2 + 25$
	& multiply out attempt			
	$AC^2 = 25p^2 - 50p + 50$	A1	2	**ag**
	$125 = 25p^2 - 50p + 50$	M1		Equating to 125 (or condone $\sqrt{125}$)
	$0 = p^2 - 2p - 3 = (p-3)(p+1)$	m1		Solving/factorising
	$\Rightarrow p = 3; \quad p = -1$	A1	3	
	Total		**12**	