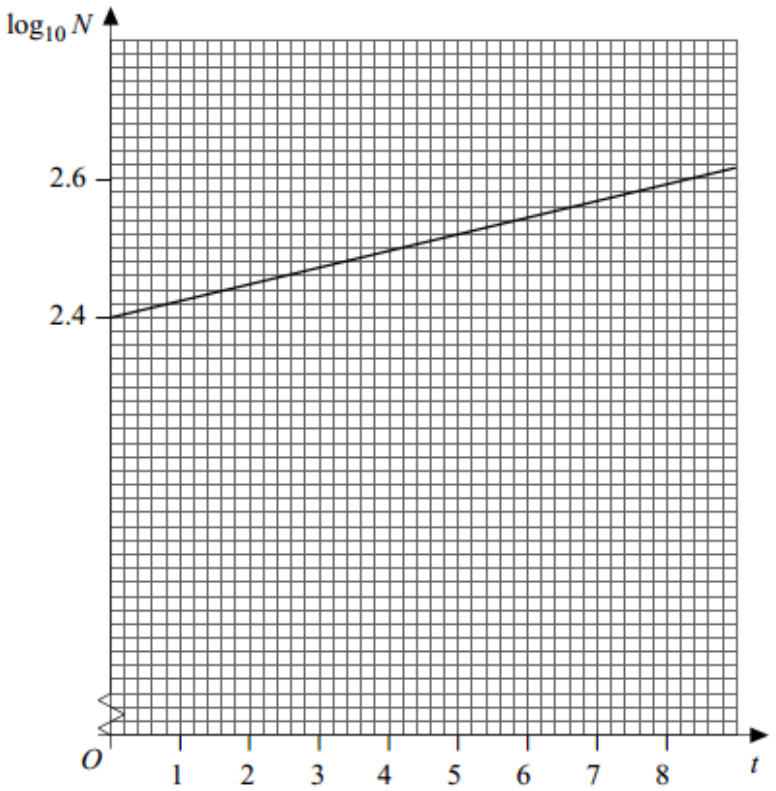
7 A mathematical model is required to estimate the number, N, of a certain strain of bacteria in a test tube at time t hours after a certain instant.

After values of $\log_{10} N$ are plotted against t, a straight line graph can be drawn through the points as shown below.



(a) Use the graph to estimate the number of bacteria when $t = 5$. *(3 marks)*

(b) The graph would suggest that N and t are related by an equation of the form

$$N = a \times b^t$$

where a and b are constants.

(i) Express $\log_{10} N$ in terms of $\log_{10} a$, $\log_{10} b$ and t. *(2 marks)*

(ii) Use the graph to determine the values of a and b, giving your answers to 3 significant figures. *(4 marks)*

(c) Suggest why the model $N = a \times b^t$ is likely to give an overestimate of the number of bacteria in the test tube for large values of t. *(1 mark)*

7(a)	2.52	B1		Seen (even if log of this value taken)
	$N = 10^{2.52}$	M1		
	= 331	A1	3	Accept 300 or 330 following correct logs
(b)(i)	$\log_{10} N = \log_{10} a + t \log_{10} b$	B2	2	B1 if ln used or $\log_{10} b^t$ not simplified
(ii)	$\log_{10} a$ is intercept on $\log_{10} N$ axis	M1		$\log_{10} a = 2.4$
	$a = 251$	A1		Must be 3sf or better
	Gradient is $\log_{10} b = \frac{0.12}{5}$ etc	M1		
	$b = 1.06$	A1	4	Must be 3sf or better May score M1 for setting up 2 equations M1 for solving one or two equations A2, 1 for correct answers
(c)	Growth limited by test tube; some die etc	E1	1	
	Total		**10**	

5 [A sheet of graph paper is provided for use in this question.]

The variables T and L satisfy a relationship of the form $T = aL^b$, where a and b are constants.

Measurements of T for given values of L gave the following results.

L	2	3	4	5	6
T	5.62	6.94	8.03	8.98	9.97

(a) Express $\ln T$ in terms of a, b and $\ln L$. *(1 mark)*

(b) Plot $\ln T$ against $\ln L$ on graph paper. *(3 marks)*

(c) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(d) Use your line to estimate

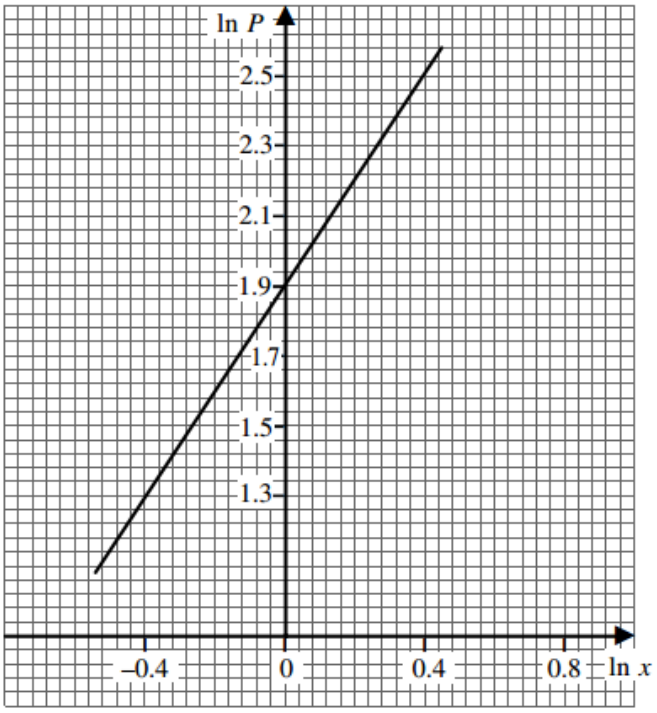
(i) the value of L when $T = 8.50$ giving your answer to two significant figures, *(2 marks)*

(ii) the values of a and b, giving your answers to two significant figures. *(4 marks)*

5 (a)	$\ln T = \ln a + b \ln L$	B1	**(1)**	
(b)	$\ln L$ 0.693 1.099 1.386			1 1.792 . 6 0 9
	$\ln T$ 1.726 1.937 2.083	B1		2 2.300 . 1 9 5
	Plotting points	M1		
	roughly correct	A1	**(3)**	
(c)	Straight line of reasonable fit	B1	**(1)**	
(d)(i)	$T = 8.50 \Rightarrow \ln T = 2.14$			
	From graph $\ln L = 1.5$	M1		must be $\ln L$
	$\Rightarrow L \approx 4.5$	A1 √	**(2)**	
(ii)	$\ln a =$ intercept	M1		proper scale
	From graph $a \approx 3.9 \leftrightarrow 4.1$	A1		
	gradient $= b$	M1		
	$\approx 0.50 \leftrightarrow 0.52$	A1	**(4)**	
		TOTAL	**(11)**	

5 A mathematical model is used by an astronomer to investigate features of the moons of a particular planet. The mean distance of a moon from the planet, measured in millions of kilometres, is denoted by x, and the corresponding period of its orbit is P days.

The model assumes that the graph of $\ln P$ against $\ln x$ is the straight line drawn below.



(a) Use the graph to estimate the period of the orbit of a moon for which $x = 1.43$. *(3 marks)*

(b) The graph would suggest that P and x are related by an equation of the form

$$P = kx^{\alpha}$$

where k and α are constants.

(i) Express $\ln P$ in terms of $\ln k$, $\ln x$ and α. *(1 mark)*

(ii) Use the graph to determine the values of k and α, giving your answers to 2 significant figures. *(4 marks)*

Q	Solution	Marks	Total	Comments
5(a)	ln 1.43 = 0.358…	M1		
	From graph ln$P$ = 2.4…	m1		Expected in range 2.43 to 2.45
	Hence P = 11.4/5/6	A1	3	Follow through their values within range
(b)(i)	ln P = ln k +α ln x	B1	1	
(ii)	ln k is intercept on vertical axis	M1		ln k = 1.9 (or use of formula)
	k = 6.7 (to 2 SF)	A1		
	Gradient of graph gives α	M1		M0 if further wrong calculation using exponentials
	α = 1.5(to 2 SF)	A1	4	
	Total		**8**	

6 [A sheet of graph paper is supplied for use in this question.]

The energy, E, lost in a cycle of magnetization of a transformer core is thought to relate to the flux density, B, by a law of the form $E = kB^{\alpha}$ where k and α are constants.

(a) Express $\ln E$ in terms of $\ln k$, α and $\ln B$. *(1 mark)*

For a given material, the values of B and E in appropriate units are:

B	3.16	9.56	18.3	29.0	41.4
E	1	2	3	4	5

(b) Plot $\ln E$ against $\ln B$ on graph paper. *(3 marks)*

(c) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(d) Use your line to estimate:

(i) the value of E when $B = 25.5$ giving your answer to 2 significant figures; *(3 marks)*

(ii) the values of k and α, giving your answers to 2 significant figures. *(4 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
6(a)	$\ln E = \ln K + \alpha \ln B$	B1	1	
(b)	$\ln B$ 1.151 2.258 2.907			3.367 3.723
	$\ln E$ 0 0.693 1.099	B2 (–1ee)		1.386 1.609
	plotting points – roughly correct	M1	3	
(c)	straight line of reasonable fit	B1	1	
(d)(i)	$B = 25.5 \Rightarrow \ln B = 3.2387$	M1		
	From graph $\ln E \approx 1.31$	M1		
	$\Rightarrow E = 3.7$	A1	3	Condone 3.6 to 3.8
(ii)	gradient $= \alpha = \frac{\Delta \ln E}{\Delta \ln B}$	M1		
	$= \frac{1.792}{2.865} \approx 0.63$	A1		Condone 0.62 to 0.64
	Intercept used/or 2 points	M1		full attempt to find k
	$k \approx 0.48$ / 0.49	A1	4	
	Total		**12**	