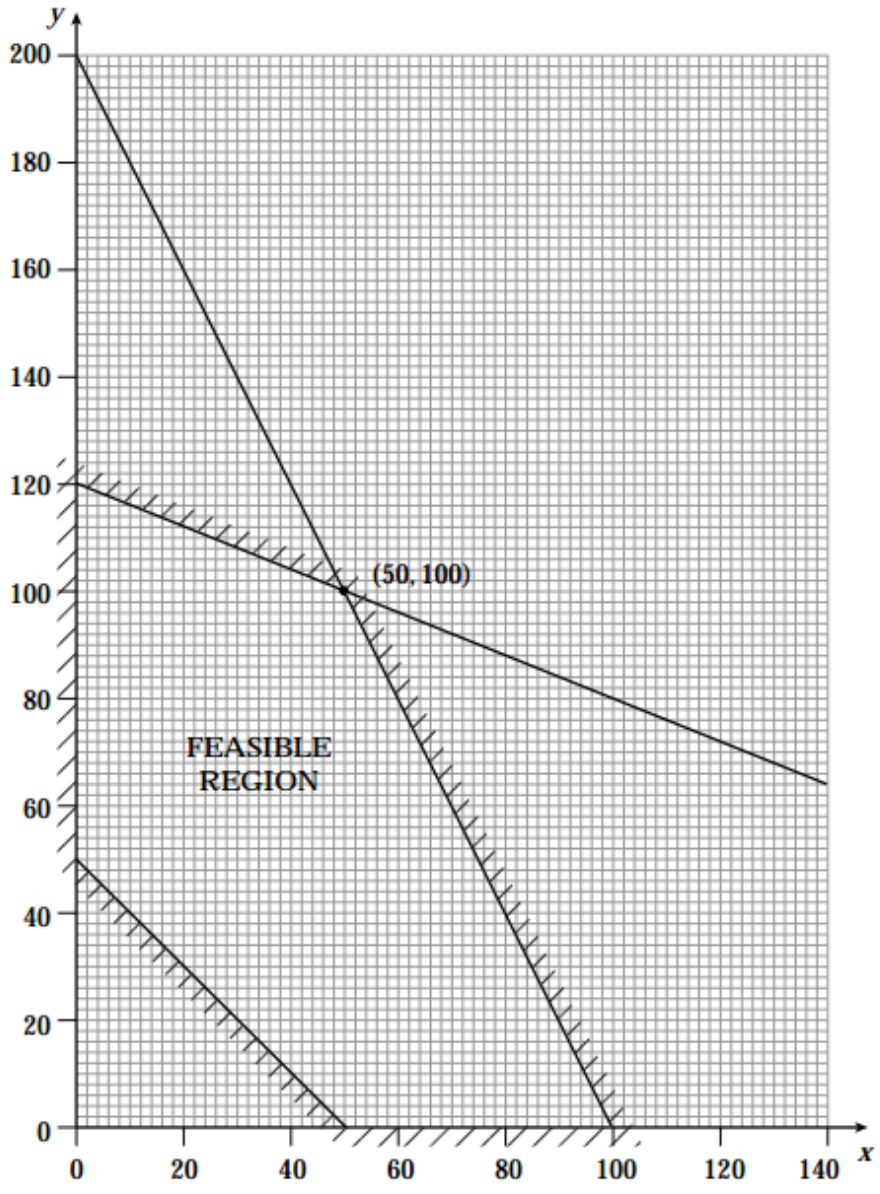
D1 Linear Programming

Challenge 1

The following graph shows the feasible region of a linear programming problem.



(a) On this feasible region find

(i) the maximum value of the function $2x + 3y$, *(3 marks)*

(ii) the minimum value of the function $4x + y$. *(2 marks)*

(b) Find the **five** inequalities that define the feasible region. *(6 marks)*

Challenge 2

[Graph paper is provided for use in this question.]
A garage owner is to restock with new cars of three different types; hatchbacks, sports cars and estate cars.

She decides that:

the number of hatchbacks must be greater than the combined total number of sports cars and estates;

the number of sports cars must not exceed 15% of the total number of cars;

there must be at least as many sports cars as estates;

the number of estates must be less than 60% of the number of hatchbacks.

Each hatchback costs £8000, each sports car costs £12000, and each estate costs £14000.

The garage owner buys x hatchbacks, y sports cars and z estates.

She wishes to minimise her total cost, £C.

(a) Formulate the above situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers. *(7 marks)*

(b) The garage owner buys 6 sports cars. By using a graphical method, find her minimum total cost in this case. *(6 marks)*

Challenge 3

[Graph paper is provided for use in answering this question.]

The Elves toy company makes toy trains and dolls' prams, which use the same wheels and logo stickers.

Each train requires 8 wheels and 2 logo stickers.

Each pram requires 8 wheels and 3 logo stickers.

The company has 7200 wheels and 2200 logo stickers available.

The company is to make at least 300 of each type of toy and at least 800 toys in total.

The company sells each train for £20 and each pram for £25.

The company makes and sells x trains and y prams.

The company needs to find its minimum and maximum total income, £T.

(a) Formulate the company's situation as a linear programming problem. *(5 marks)*

(b) Draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(c) Use your diagram to find the company's minimum and maximum total income, £T. *(4 marks)*

Final Challenge

[**Figure 2**, printed on a separate sheet, is provided for use in answering this question.]

The Tony television company makes analogue and digital televisions. Both types of television require a number of component *A* and component *B*.

Each analogue television requires 2 of component *A* and 3 of component *B*.
Each digital television requires 4 of component *A* and 1 of component *B*.

Each day:
the company has 50 of component *A* and 24 of component *B* available; and
the company is to make at least 2 of each type of television, but no more than 20 in total.

The company sells each analogue television at a profit of £20 and each digital television at a profit of £25.
Each day, the company makes and sells *x* analogue and *y* digital televisions.
The company needs to find its minimum and maximum daily income, £*T*.

(a) Formulate the company's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(c) Use your diagram to find the company's minimum and maximum daily income, £*T*. *(6 marks)*

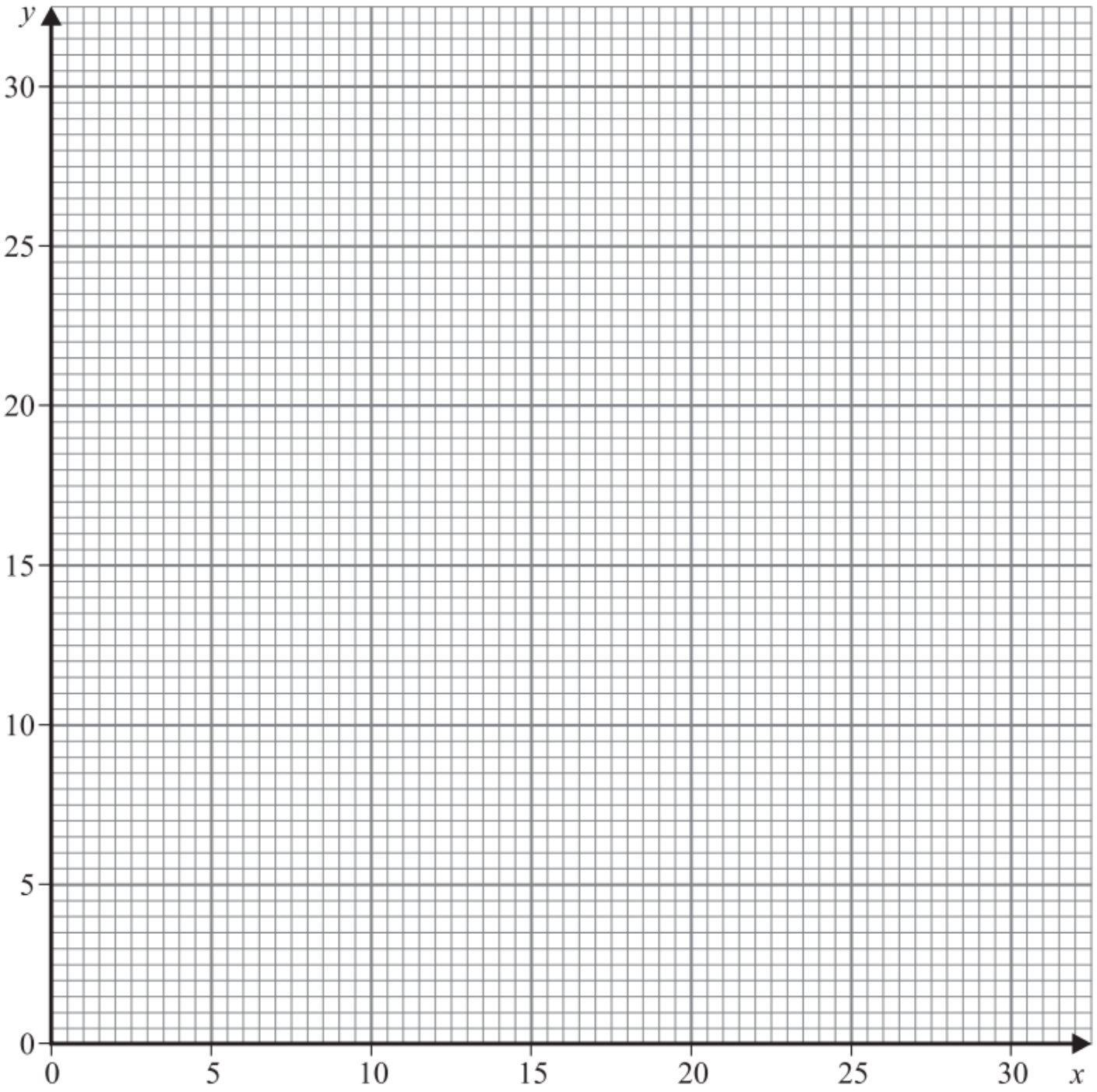


Figure 2