1 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{AB}$. *(2 marks)*

(b) Show that $\mathbf{A} + \mathbf{B} - \mathbf{AB} = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

Q	Solution	Marks	Total	Comments
1(a)(i)	$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(ii)	$\mathbf{AB} = \begin{bmatrix} 8 & 6 \\ 6 & 8 \end{bmatrix}$	M1A1	2	Ditto
(b)	$\mathbf{A} + \mathbf{B} - \mathbf{AB} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$	B1F		ft wrong answers in (a)
	$\ldots = -5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1	2	
	Total		**6**	

1 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

(i) **AB** ; *(2 marks)*

(ii) **ABC** . *(2 marks)*

(b) Describe the geometrical transformation represented by the matrix **AB**. *(2 marks)*

Question	Solution	Marks	Total	Comments
1(a)(i)	$\mathbf{AB} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$	M1A1	2	M1 for two correct entries
(ii)	$\mathbf{ABC} = \begin{bmatrix} 30 & 20 \\ 10 & 0 \end{bmatrix}$	M1		ditto
		A1F	2	ft wrong answer to (i)
(b)	Enlargement ...	M1		
	... with scale factor 10	A1	2	
	Total		**6**	

2 The matrix $\boldsymbol{M}$ is $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$.

(a) Find:

(i) $\boldsymbol{M}^2$; *(2 marks)*

(ii) $\boldsymbol{M}^3$. *(1 mark)*

(b) The transformation $\mathbf{T}$ is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Describe fully the geometrical transformation represented by $\mathbf{T}$. *(2 marks)*

2(a)	$M^2 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	M1 A1		Attempt to multiply matrices correctly Correct
	$M^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	A1	3	
(b)	Rotation (about origin)	M1		
	through $\frac{2\pi}{3}$ (anticlockwise)	A1	2	Or equivalent clockwise turn
	Total		**5**	

4 A transformation T_1 is represented by the matrix

$$\mathbf{M}_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

(a) Give a geometrical description of T_1. *(3 marks)*

The transformation T_2 is a reflection in the line $y = \sqrt{3}x$.

(b) Find the matrix $\mathbf{M}_2$ which represents the transformation T_2. *(3 marks)*

(c) (i) Find the matrix representing the transformation T_2 followed by T_1. *(2 marks)*

(ii) Give a geometrical description of this combined transformation. *(3 marks)*

Q	Solution	Marks	Total	Comments
4 (a)	Rotation, $\frac{\pi}{6}$, anticlockwise	B1B1B1	3	
(b)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	B3	3	B2 if 2 correct
(c)(i)	$\mathbf{M}_1\mathbf{M}_2$ considered	M1		
	$\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	A1	2	
(ii)	Reflection	B1		
	Line at 75° to x - axis	B2	3	
	Total		**11**	