FP1 Matrices Challenge

Challenge 1

The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

   (i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

   (ii) $\mathbf{AB}$. *(2 marks)*

(b) Show that $\mathbf{A} + \mathbf{B} - \mathbf{AB} = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

Challenge 2

The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

(a) Calculate the matrices:

    (i) **AB** ;   *(2 marks)*

    (ii) **ABC** .   *(2 marks)*

(b) Describe the geometrical transformation represented by the matrix **AB**.   *(2 marks)*

Challenge 3

The matrix M is $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$.

(a) Find:

    (i) M^2;   *(2 marks)*

    (ii) M^3.   *(1 mark)*

(b) The transformation T is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$$

    Describe fully the geometrical transformation represented by T.   *(2 marks)*

Final Challenge

A transformation T_1 is represented by the matrix

$$\mathbf{M}_1 = \begin{bmatrix} \dfrac{\sqrt{3}}{2} & -\dfrac{1}{2} \\[2mm] \dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \end{bmatrix}.$$

(a)   Give a geometrical description of T_1.   (3 marks)

The transformation T_2 is a reflection in the line $y = \sqrt{3}x$.

(b)   Find the matrix $\mathbf{M}_2$ which represents the transformation T_2.   (3 marks)

(c)   (i)   Find the matrix representing the transformation T_2 followed by T_1.   (2 marks)

(ii)   Give a geometrical description of this combined transformation.   (3 marks)