3 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix}.$$

(a) Give a geometrical description of each of the transformations represented by the matrices **A** and **B**. *(6 marks)*

(b) For each of these transformations, find the line of invariant points. *(2 marks)*

3 (a)	**A** Shear Parallel to y-axis $(1, 0) \to (1, 3)$	M1 A1 B1		e.g.(check suggested point) $(1, 1) \to (1, 4)$ **not** SF
	B Rotation About x-axis of 45°	M1 A1 A1	6	or "in y-z plane"
(b)	**A** y-axis	B1		or $x = 0$, $\begin{bmatrix} 0 \\ \lambda \end{bmatrix}$
	B x-axis	B1	2	or $y = z = 0$, $\begin{bmatrix} \lambda \\ 0 \\ 0 \end{bmatrix}$
	Total		**8**	

1 (a) The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Find the matrix **AB**. *(2 marks)*

(b) The matrix **M** is given by

$$\mathbf{M} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(i) Give a geometrical description of the transformation represented by the matrix **M**. *(4 marks)*

(ii) Which line is invariant under **M**? *(1 mark)*

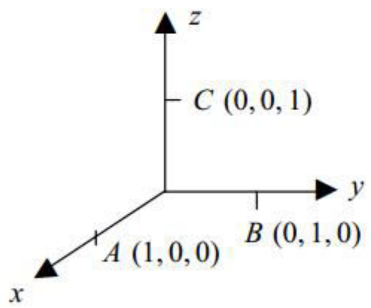
Q	Solution	Marks	Total	Comments
1(a)	$\mathbf{AB} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	B2,1,0	2	
(b)(i)	This transformation represents a rotation of 180°	B1		accept reflection in z-axis or in both x- and y-axes
	about the z-axis	B1		
	together with an enlargement scale factor 3	B1		accept 'stretch' as long as clear, but not enlargement along an axis
	from the origin	B1	4	
(ii)	z-axis	B1	1	OE
	Total		**7**	

3 A matrix $\mathbf{M}_1$ is given by

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Give a geometrical description of the transformation T_1 represented by $\mathbf{M}_1$. *(2 marks)*

(b) The diagram below shows the points $A\ (1, 0, 0)$, $B\ (0, 1, 0)$ and $C\ (0, 0, 1)$.



A second transformation, T_2, is a rotation of π radians about the line $x = z$, $y = 0$.

(i) Find the images of the points A, B, and C under T_2. *(2 marks)*

(ii) Write down the matrix $\mathbf{M}_2$ which represents this transformation. *(2 marks)*

(c) (i) Show that the matrix $\mathbf{M}_3$, which represents the transformation T_2 followed by the transformation T_1, is given by

$$\mathbf{M}_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ *(2 marks)*

(ii) Give a geometrical description of the transformation represented by the matrix $\mathbf{M}_3$. *(2 marks)*

Q	Solution	Marks	Total	Comments
3 (a)	$\mathbf{M}_1$ is a rotation of $-\frac{\pi}{2}$ about y-axis	B1B1	2	Accept $-\frac{\pi}{2}$, 90°
(b)(i)	$(1,\ 0,\ 0) \to (0,\ 0,\ 1)$ $(0,\ 1,\ 0) \to (0, -1,\ 0)$ $(0,\ 0,\ 1) \to (1,\ 0,\ 0)$	B2,1,0	2	
(ii)	Matrix $\mathbf{M}_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	M1A1F	2	
(c)(i)	$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1A1	2	AG M1 for getting the order of the matrices correct
(ii)	Rotation of π about the z-axis	B1B1	2	Accept 180°
	Total		**10**	

4 Three linear transformations, T_1, T_2 and T_3, of three-dimensional space are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{M}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad \text{and} \quad \mathbf{M}_3 = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 2 & 6 \\ 2 & 4 & 4 \end{bmatrix}$$

respectively.

(a) Give a geometrical description of the transformations

(i) T_1, *(2 marks)*

(ii) T_2. *(3 marks)*

(b) Show that the line with Cartesian equations

$$\frac{x}{4} = -\frac{y}{3} = \frac{z}{2}$$

is invariant under T_3. *(5 marks)*

Q	Solution	Marks	Total	Comments
4 (a)(i)	Reflection, (plane) $z = 0$	B1B1	2	
(ii)	Rotation, 30°, x-axis	B1B1B1	3	
(b)	$(4, -3, 2)$ $\rightarrow (8, -6, 4)$ Doubled or multiple Convincing argument	B1 M1A1 E1 +E1	5	e.g. refer to through origin
	Alternative: $(4\lambda, -3\lambda, 2\lambda)$ $\rightarrow (8\lambda, -6\lambda, 4\lambda)$ on same line	M1A1 M1A1 E1		
	Total		**10**	