FP4 - Matrix Transformations Challenge

Challenge 1

The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° \\ 0 & \sin 45° & \cos 45° \end{bmatrix}.$$

(a) Give a geometrical description of each of the transformations represented by the matrices **A** and **B**. *(6 marks)*

(b) For each of these transformations, find the line of invariant points. *(2 marks)*

Challenge 2

(a) The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Find the matrix **AB**. *(2 marks)*

(b) The matrix **M** is given by

$$\mathbf{M} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(i) Give a geometrical description of the transformation represented by the matrix **M**. *(4 marks)*

(ii) Which line is invariant under **M**? *(1 mark)*

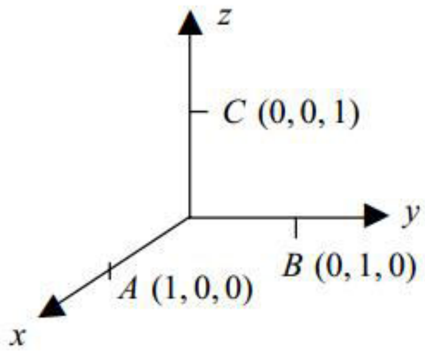
Challenge 3

A matrix $\mathbf{M}_1$ is given by

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Give a geometrical description of the transformation T_1 represented by $\mathbf{M}_1$. *(2 marks)*

(b) The diagram below shows the points $A\ (1, 0, 0)$, $B\ (0, 1, 0)$ and $C\ (0, 0, 1)$.



A second transformation, T_2, is a rotation of π radians about the line $x = z$, $y = 0$.

(i) Find the images of the points A, B, and C under T_2. *(2 marks)*

(ii) Write down the matrix $\mathbf{M}_2$ which represents this transformation. *(2 marks)*

(c) (i) Show that the matrix $\mathbf{M}_3$, which represents the transformation T_2 followed by the transformation T_1, is given by

$$\mathbf{M}_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ *(2 marks)*

(ii) Give a geometrical description of the transformation represented by the matrix $\mathbf{M}_3$. *(2 marks)*

Final Challenge

Three linear transformations, T_1, T_2 and T_3, of three-dimensional space are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{M}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad \text{and} \quad \mathbf{M}_3 = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 2 & 6 \\ 2 & 4 & 4 \end{bmatrix}$$

respectively.

(a) Give a geometrical description of the transformations

(i) T_1, *(2 marks)*

(ii) T_2. *(3 marks)*

(b) Show that the line with Cartesian equations

$$\frac{x}{4} = -\frac{y}{3} = \frac{z}{2}$$

is invariant under T_3. *(5 marks)*