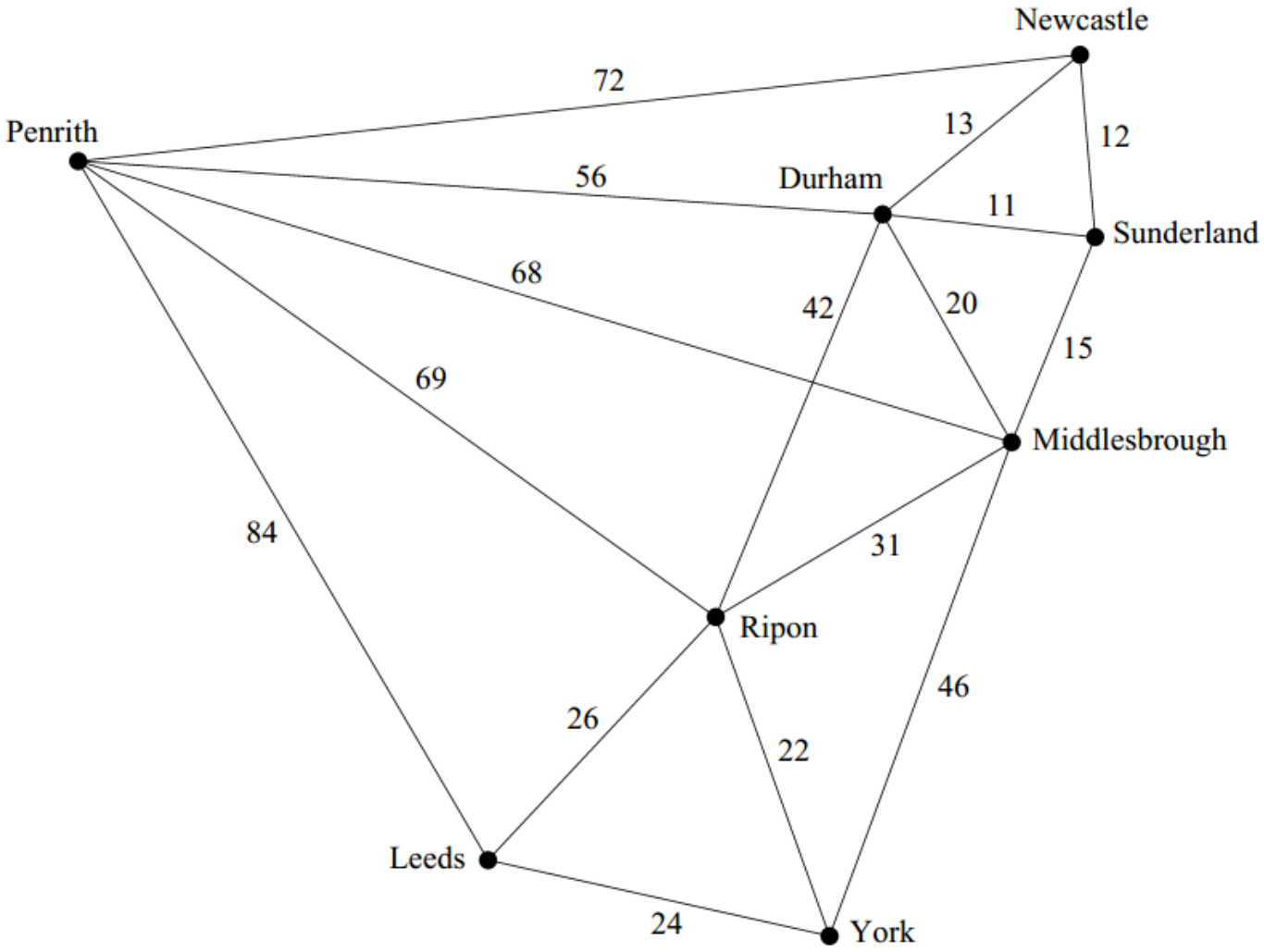
D1 Minimum Connector

Challenge 1

2 The following diagram shows a network of roads connecting eight towns. The number on each arc represents the distance, in miles, between two towns.

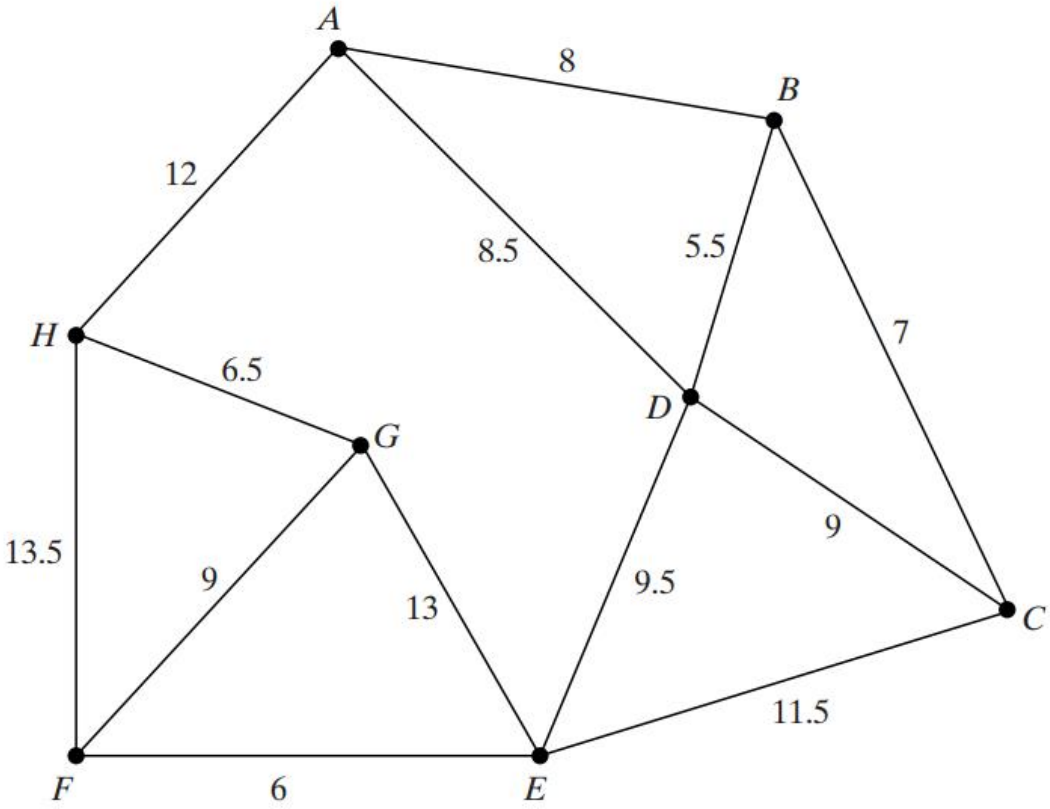


(a) Starting from Ripon and showing your working at each stage, use Prim's algorithm to find the minimum spanning tree for the eight towns. State the length of your minimum spanning tree. *(5 marks)*

(b) Draw your minimum spanning tree. *(1 mark)*

Challenge 2

The following diagram shows the lengths of roads, in miles, connecting eight towns.



Using Kruskal's algorithm, showing your working at each stage, find the minimum spanning tree for the network. State its length. *(6 marks)*

Challenge 3

[Figure 2, printed on the insert, is provided for use in answering this question.]

A world-wide family consists of people **A**, **B**, **C**, **D**, **E**, **F** and **G**. The minimum cost of a phone-call, in pence, between any pair of them is shown in the following table:

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	-	80	20	95	10	20	10
B	80	-	70	45	60	70	80
C	20	70	-	80	60	25	25
D	95	45	80	-	70	80	90
E	10	60	60	70	-	25	15
F	20	70	25	80	25	-	30
G	10	80	25	90	15	30	-

Person **A** wishes to pass a piece of news to all the other family members, either by a direct phone-call or by the message being passed on by other phone calls.

(a) By applying Prim's algorithm to the matrix in Figure 2 on the insert (or otherwise), find the minimum cost of notifying the whole family of the news. *(5 marks)*

(b) Person **A** joins a telephone discount scheme which halves the cost of all phone calls to and from him. Without repeating the whole algorithm, state how this would affect your answer to part (a) and calculate the new minimum cost of notifying the whole family. *(4 marks)*

Final Challenge

[**Figure 2**, printed on the insert, is provided for use in answering part (a) of this question.]

The table shows the distances, in miles, of any direct routes between the seven towns A, B, C, D, E, F and G:

	A	B	C	D	E	F	G
A	–	5	15	–	–	–	7
B	5	–	8	–	–	9	15
C	15	8	–	7	13	–	6
D	–	–	7	–	5	6	10
E	–	–	13	5	–	5	–
F	–	9	–	6	5	–	6
G	7	15	6	10	–	6	–

(a) Use Prim's algorithm to find a minimum connector of the seven towns, and state its length. *(5 marks)*

(b) The local authority decides to charge tolls on some of these roads, but it must still be possible to travel between any two towns on toll-free roads. What is the maximum length of roads on which it can charge tolls? *(2 marks)*

(c) Opposition from local residents forces the authority to keep the road from B to G free of tolls (and to leave enough other toll-free roads so that it is still possible to travel between any two towns on toll-free roads). With this extra restriction, what is the maximum length of roads on which the local authority can charge tolls? *(3 marks)*

Figure 2 for Challenge 3

	A	B	C	D	E	F	G
A	-	80	20	95	10	20	10
B	80	-	70	45	60	70	80
C	20	70	-	80	60	25	25
D	95	45	80	-	70	80	90
E	10	60	60	70	-	25	15
F	20	70	25	80	25	-	30
G	10	80	25	90	15	30	-

Figure 2 for Final Challenge

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	5	15	–	–	–	7
B	5	–	8	–	–	9	15
C	15	8	–	7	13	–	6
D	–	–	7	–	5	6	10
E	–	–	13	5	–	5	–
F	–	9	–	6	5	–	6
G	7	15	6	10	–	6	–