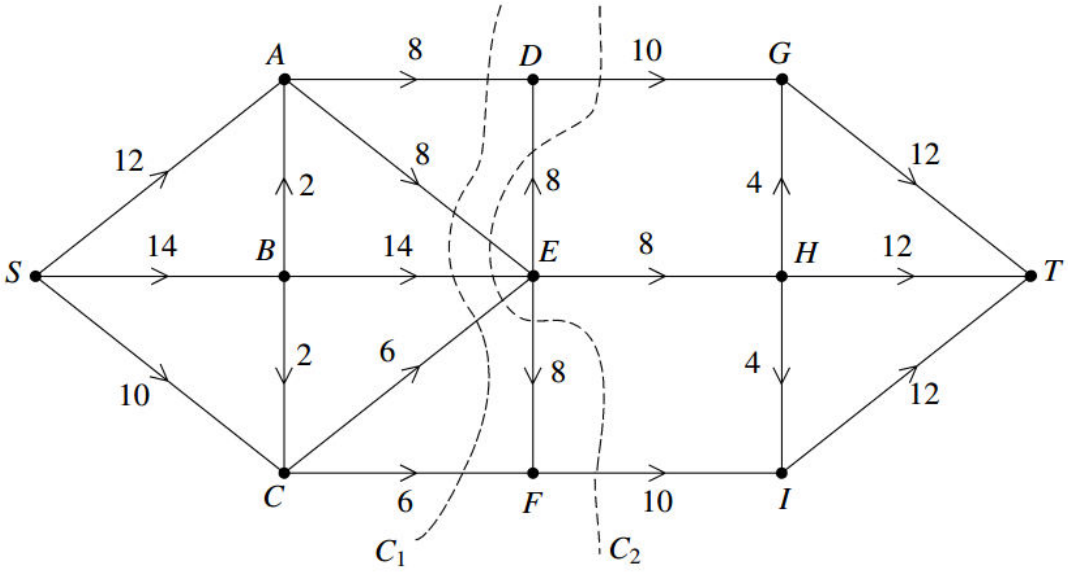
D2 Network flows Challenge

Challenge 1

[**Figure 1**, printed on the insert, is provided for use in answering this question.]

The following diagram shows a network of drainage pipes on a housing estate. Vertex S is the highest point on the estate and vertex T is the lowest point on the estate. The numbers on the edges show the maximum volume of water, in cubic metres, that can flow along each section of the pipe in one minute.



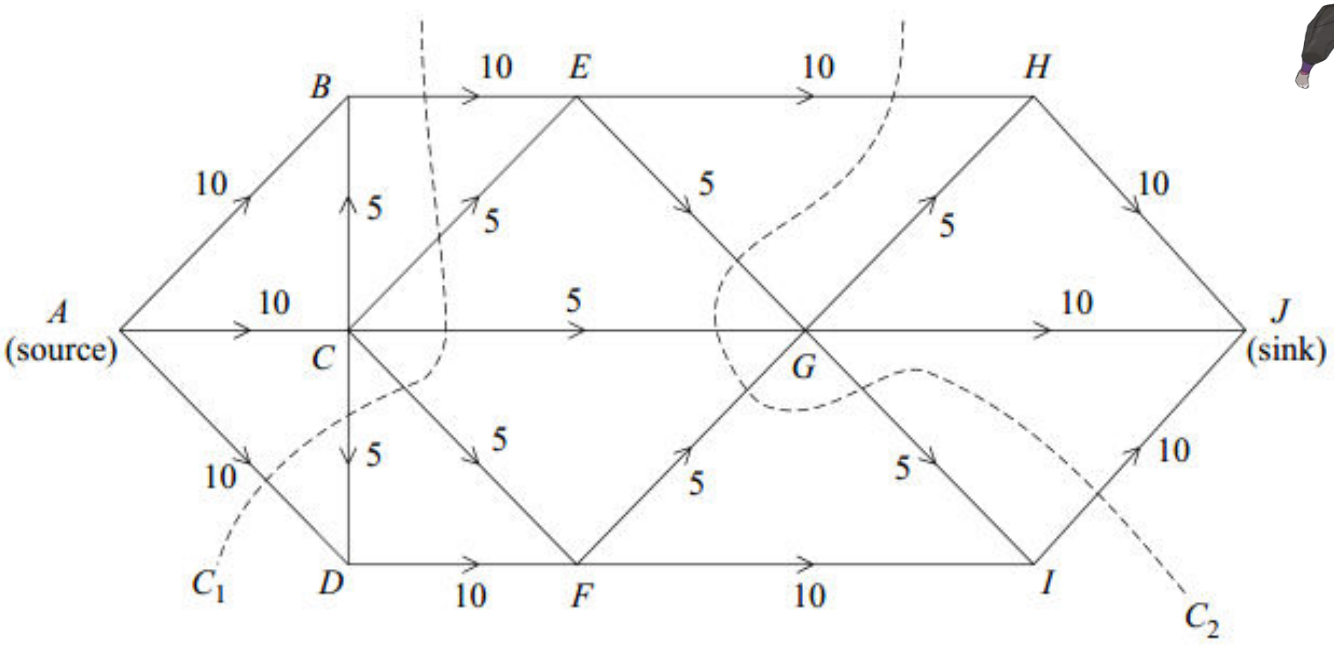
(a) Find the value of the cuts marked C_1 and C_2 on the diagram. *(3 marks)*

(b) On **Figure 1**, starting from a position of zero flow, use flow augmentation to find the maximum flow of water through the drainage pipes. *(7 marks)*

Challenge 2

[Figures 2, 3 and 4, printed on the insert, are provided for use in answering this question.]

The following diagram shows ten vertices connected by a number of arcs. The numbers on the arcs represent the maximum flow along each arc.



(a) Find the values of the cuts C_1 and C_2. *(2 marks)*

(b) Find, by inspection, a flow of 30 units and illustrate it on **Figure 2**. *(2 marks)*

(c) On a certain day, the section GJ is blocked so that the network is now as indicated on **Figure 3**.

Find, by inspection, the maximum flow on this day and illustrate it on **Figure 3**. *(2 marks)*

(d) On another day, GJ is working normally but EH is now blocked, as shown on **Figure 4**.

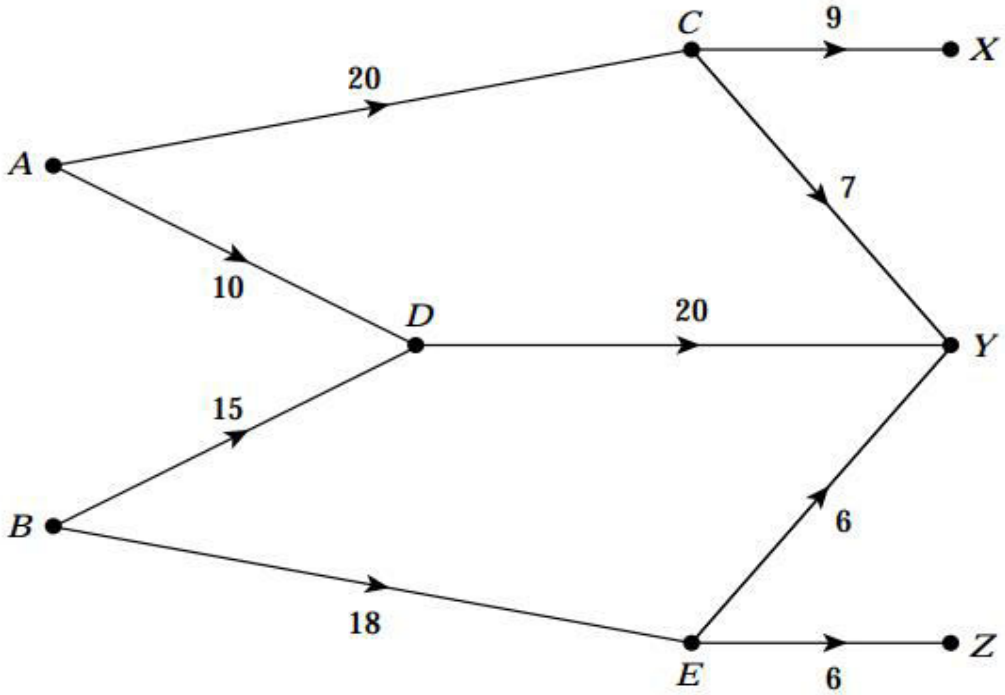
(i) Find, by inspection, the maximum flow on this day and illustrate it on **Figure 4**. *(2 marks)*

(ii) Show that the flow obtained in part (d)(i) is maximal. *(2 marks)*

Final Challenge

[**Figures 2** and **3**, printed on a separate sheet, are provided for use in answering this question.]

A greengrocer has two suppliers, A and B, and three storage depots, C, D and E. He needs to transport his stock to three retail outlets X, Y and Z. The capacities of the possible routes, in van loads per week, are shown in the following diagram.



(a) Add a super-source (S) and a super-sink (W) on **Figure 2** to obtain a single source, single sink capacitated network. Show the capacities of each arc you have added. *(2 marks)*

(b) State the maximum flow along the routes $SADYW$ and $SBEZW$. *(2 marks)*

(c) (i) Show your answers to part (b) on **Figure 3** and, taking this as the initial flow pattern, use flow augmentation to find the maximum flow from S to W. *(6 marks)*

(ii) Prove that your flow is maximal. *(2 marks)*

Network flow inserts

Challenge 1

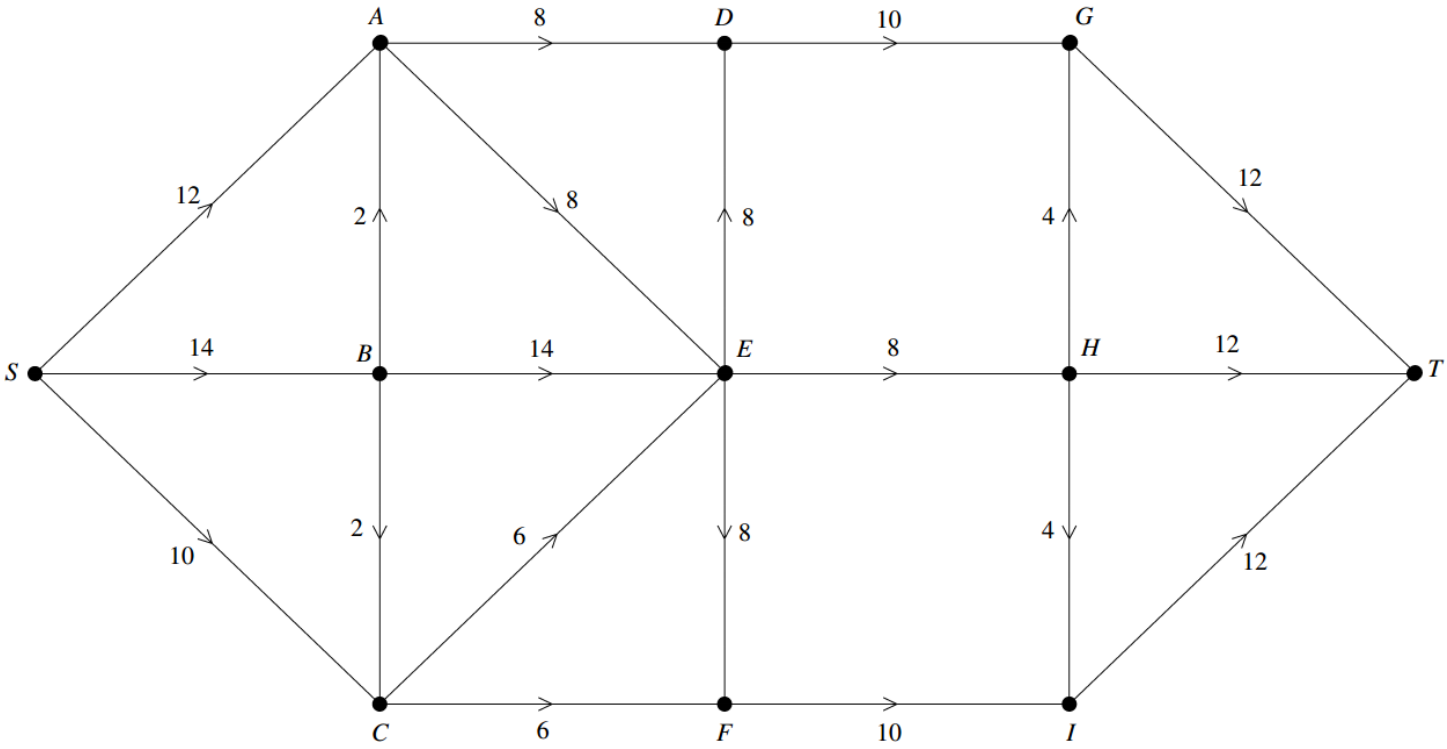


Figure 1 for use in Question 3

Challenge 2

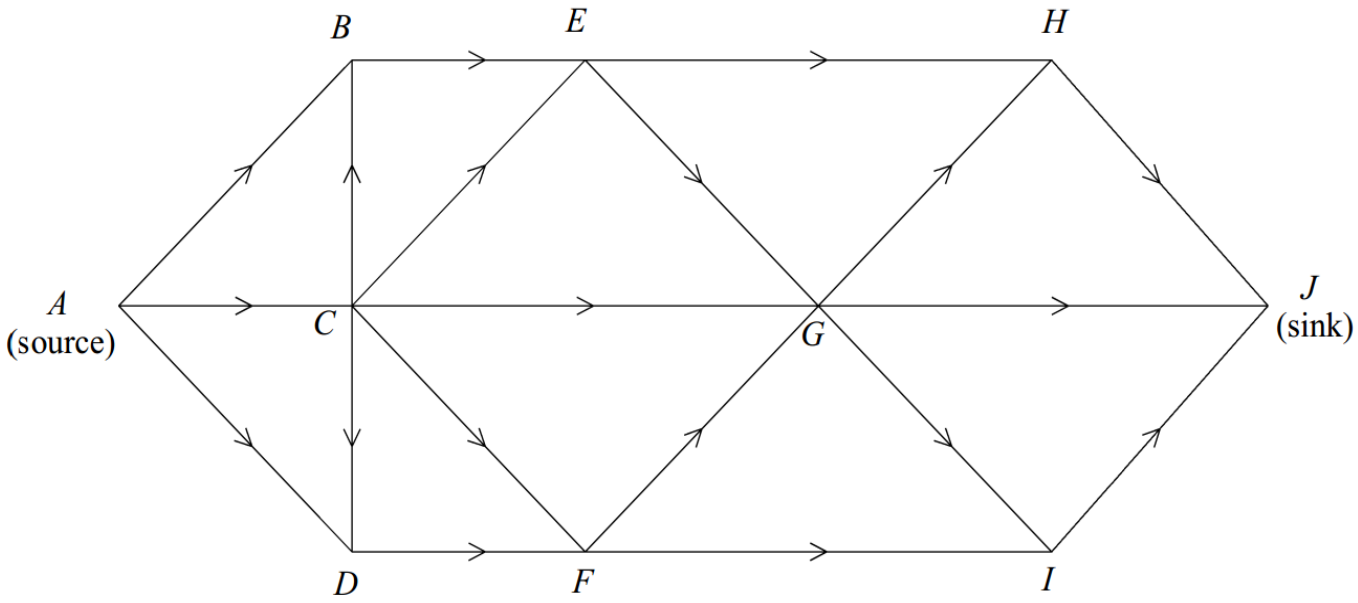


Figure 2 (for Question 4(b))

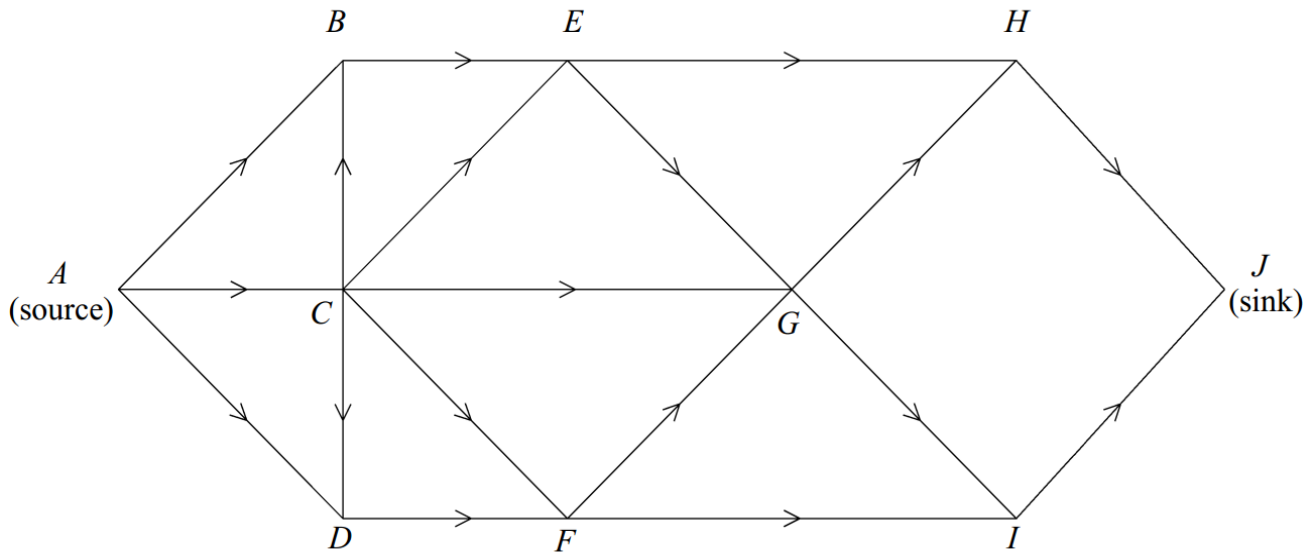


Figure 3 (for Question 4(c))

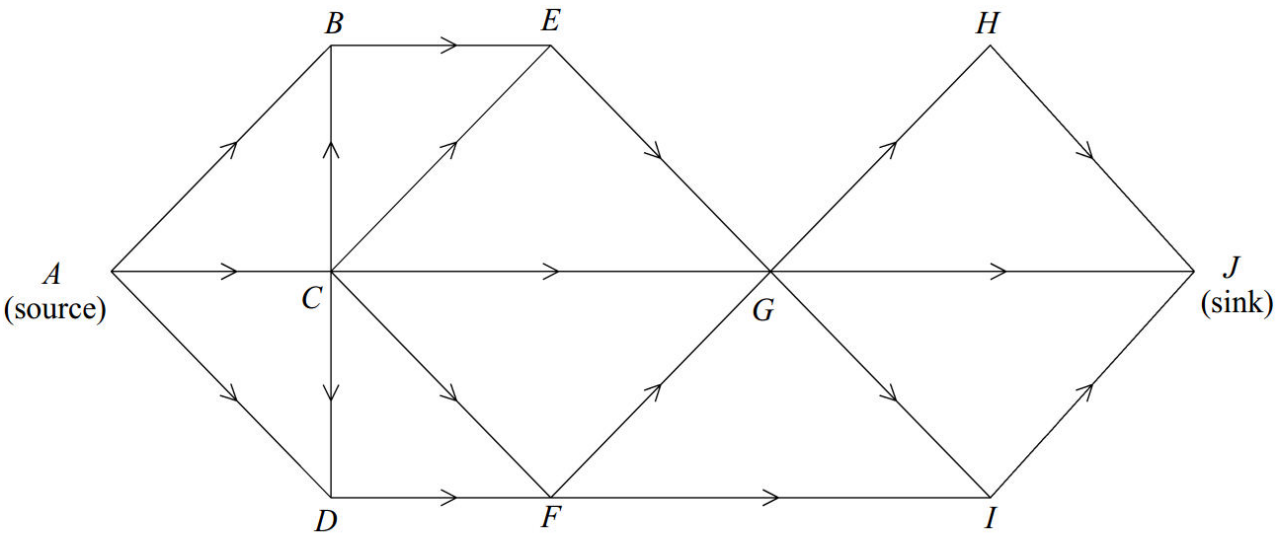


Figure 4 (for Question 4(d))

Challenge 3