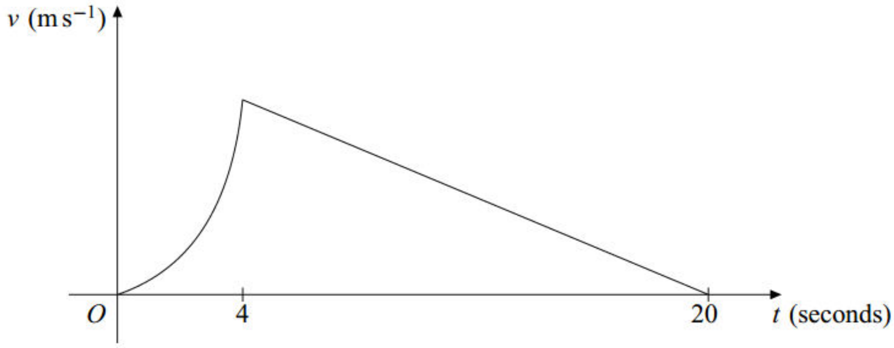
7



The velocity-time graph shows the velocity, $v \text{ m s}^{-1}$, at time t seconds, of a particle P which moves in a straight line. The motion of P has two separate stages.

In the first stage, P moves with velocity $v = 2t^2$ for 4 seconds.

In the second stage, P moves with a constant retardation for a further 16 seconds until coming to rest.

(a) Find the value of v when $t = 4$. *(1 mark)*

(b) Find the total distance travelled by P during the **two** stages of the motion. *(6 marks)*

(c) The particle has mass 0.2 kg. Find the magnitude of the force acting on P when:

(i) $t = 2$; *(4 marks)*

(ii) $t = 10$. *(3 marks)*

7(a)	$v = 32$	B1	1	
(b)	$S_1 = \int_0^4 2t^2\,dt$	M1		Attempt to integrate
	$= \left[\frac{2t^3}{3}\right]_0^4$	A1		
	$= 2\times\frac{64}{3} - 0$	m1		Both limits used or *c* found
	$S_2 = \frac{1}{2}\times 16\times 32$	M1		
	$= 256$	A1F		
	Total distance = 299	A1F	6	AWRT
(c)(i)	acc = $4t$	M1		
	$= 4\times 2$	A1		
	Force = m × acc	m1		Used
	= 1.6 N	A1F	4	
(ii)	$\text{acc} = \frac{0-32}{16}$	M1		
	acc/ret = ± 2	A1		
	$\lvert\text{force}\rvert = +\,0.4$ N	A1F	3	Condone +2 used throughout if deceleration or retardation stated
	Total		**14**	

6 A particle moves so that, at time t, its acceleration is $3e^{-2t}\mathbf{i} + 2\mathbf{j}$. Its initial velocity is $5\mathbf{i}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular.

Show that the velocity $\mathbf{v}$ of the particle at time t is given by

$$\mathbf{v} = \left(\frac{13 - 3e^{-2t}}{2}\right)\mathbf{i} + 2t\mathbf{j}$$

(7 marks)

Question Number and part	Solution	Marks	Total Marks	Comments
6	$v = \int 3e^{-2t}\,dt\,\mathbf{i} + \int 2\,dt\,\mathbf{j}$	M1A1		Integrating $3e^{-2t}$
	$= \left(-\frac{3}{2}e^{-2t} + c\right)\mathbf{i} + (2t + d)\mathbf{j}$	M1 A1		Integrating 2 Constants of integration
	$d = 0$	B1		$d = 0$
	$5 = -\frac{3}{2} + c$			
	$c = \frac{13}{2}$			
	$\mathbf{v} = \left(\frac{13 - 3e^{-2t}}{2}\right)\mathbf{i} + 2t\mathbf{j}$	M1 A1	7	Finding c Correct c plus correct final form
	Total		**7**	

7 A particle has mass 2000 kg. A single force, $\mathbf{F} = 1000t\mathbf{i} - 5000\mathbf{j}$ newtons, acts on the particle, at time t seconds. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular. No other forces act on the particle.

(a) Find an expression for the acceleration of the particle. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $6\mathbf{j}\,\mathrm{m\,s^{-1}}$. Show that at time t the velocity, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, of the particle is given by

$$\mathbf{v} = \frac{t^2}{4}\mathbf{i} + \left(6 - \frac{5t}{2}\right)\mathbf{j}$$ *(4 marks)*

(c) The particle is initially at the origin. Find an expression for the position vector, $\mathbf{r}$ metres, of the particle at time t seconds. *(4 marks)*

Q	Solution	Marks	Total	Comments
7(a)	$\mathbf{a} = \frac{1000t\mathbf{i} - 5000\mathbf{j}}{2000} = \frac{t}{2}\mathbf{i} - \frac{5}{2}\mathbf{j}$	M1 A1	2	Using Newton's 2nd law Correct acceleration
(b)	$\mathbf{v} = \left(\frac{t^2}{4} + c\right)\mathbf{i} + \left(\frac{-5t}{2} + d\right)\mathbf{j}$	M1 A1		Integration Correct integral with or without constants
	$t = 0, \mathbf{v} = 6\mathbf{j} \Rightarrow c = 0, d = 6$			
	$\mathbf{v} = \left(\frac{t^2}{4}\right)\mathbf{i} + \left(6 - \frac{5t}{2}\right)\mathbf{j}$	m1 A1	4	Evaluation of constants Correct constants and correct final answer
(c)	$\mathbf{r} = \left(\frac{t^3}{12} + e\right)\mathbf{i} + \left(6t - \frac{5t^2}{4} + f\right)\mathbf{j}$	M1 A1		Integration Correct integral with or without constants
	$t = 0, \mathbf{r} = 0\mathbf{i} + 0\mathbf{j} \Rightarrow e = 0, f = 0$			
	$\mathbf{r} = \left(\frac{t^3}{12}\right)\mathbf{i} + \left(6t - \frac{5t^2}{4}\right)\mathbf{j}$	m1 A1	4	Evaluation of constants Correct constants and correct final answer
	Total		**10**	

7 A particle P moves so that at time t seconds its position vector, $\mathbf{r}$ metres, is

$$\mathbf{r} = \begin{bmatrix} 2t^2 + 6 \\ 5t \end{bmatrix}, \qquad 0 \leqslant t \leqslant 5.$$

(a) Find the velocity of P at time t. *(2 marks)*

(b) The force acting on P is $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ newtons. Find the mass of P. *(3 marks)*

(c) At the instant when $t = 5$, an additional force, $\begin{bmatrix} 0 \\ t \end{bmatrix}$ newtons, begins to act on P.

(i) Find the resultant acceleration of P. *(3 marks)*

(ii) Find the velocity of P when $t = 10$. *(5 marks)*

Q	Solution	Marks	Total	Comments
7 (a)	$\dot{r} = \begin{bmatrix} 4t \\ 5 \end{bmatrix}$	M1A1	2	Attempt to differentiate for M1, ie one term clearly differentiated
(b)	$\ddot{r} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$	B1F		
	$F = m\ddot{r} \Rightarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix} = m\begin{bmatrix} 4 \\ 0 \end{bmatrix}$	M1		two vectors
	$\underline{m = 0.5}$	A1F	3	FT one slip, provided m is scalar
(c)(i)	$\mathbf{F}_2 = \begin{bmatrix} 0 \\ t \end{bmatrix}$, resultant $\mathbf{F} = \begin{bmatrix} 2 \\ t \end{bmatrix}$	B1		Accept $\begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ t \end{bmatrix}$
	$\begin{bmatrix} 2 \\ t \end{bmatrix} = \frac{1}{2}\mathbf{a}, \quad \Rightarrow \mathbf{a} = \begin{bmatrix} 4 \\ 2t \end{bmatrix}$	M1A1F	3	Use of $F=ma$ for M1 with candidate's F and m, A1F for function of t
(ii)	$\mathbf{v} = \int \begin{bmatrix} 4 \\ 2t \end{bmatrix} \mathrm{d}t$	M1		Can gain part (i) marks at this stage M1 for attempt to integrate, one term clearly integrated
	$= \begin{bmatrix} 4t \\ t^2 \end{bmatrix} (+\mathbf{c})$	A1F		Integration of a function of t
	$t=5, \quad \mathbf{v} = \begin{bmatrix} 20 \\ 5 \end{bmatrix}$	B1F		FT variable velocity from (a)
	$\begin{bmatrix} 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 25 \end{bmatrix} + \mathbf{c}$	m1		Find **c** or use limits on both sides ($t=5$, $\mathbf{v}=\begin{bmatrix} 20 \\ 5 \end{bmatrix}$)
	$\mathbf{c} = \begin{bmatrix} 0 \\ -20 \end{bmatrix}$			
	$\mathbf{v} = \begin{bmatrix} 40 \\ 100 \end{bmatrix} + \begin{bmatrix} 0 \\ -20 \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$	A1F	5	
	Total		**13**	