2 (a) Use logarithms to solve the equation $2^x = 7$, giving your answer to three significant figures. *(2 marks)*

(b) The equation

$$2^x = 7 - x$$

has a single root, α.

(i) Show that α lies between 2.0 and 2.4. *(1 mark)*

(ii) Use the bisection method to find an interval of width 0.1 in which α lies. *(3 marks)*

2(a)	$x \ln 2 = \ln 7$	M1		May use $\log_{10}$
	$\Rightarrow x = 2.81$	A1	2	2.80735… Accept more than 3 SF
(b) (i)	$f(x) = 2^x - 7 + x$;			
	$f(2.0) = -1$; $f(2.4) = 0.678\ldots$			
	$\Rightarrow$ root lies in interval (2.0, 2.4)	B1	1	Or equivalent considering both sides but must contain a valid conclusion
(ii)	Considering f(2.2) **first**	M1		M0 if bisection method NOT used
	$f(2.2) = -0.2052\ldots$			
	$\Rightarrow$ root lies in interval (2.2, 2.4)	A1		
	$f(2.3) = 0.224\ldots$			
	$\Rightarrow$ root lies in interval (2.2, 2.3)	A1	3	SC1 if correct interval given but bisection method not used
	Total		**6**	

6 A curve satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{9 - x^2}$.

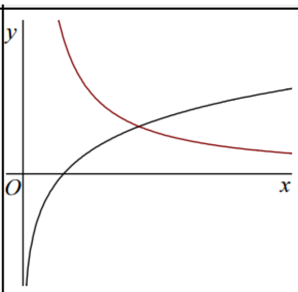
Starting at the point $(0, 3)$ on the curve, use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 1$, giving your answer to two decimal places. *(5 marks)*

Question	Solution	Marks	Total	Comments
6	$x = 0 \Rightarrow y' = 3$	M1		
	$\delta y \approx 3\delta x = 1.5$	m1		
	$x = 0.5 \Rightarrow y \approx 3 + 1.5 = 4.5$	A1		
	... and $y' \approx \sqrt{8.75} \approx 2.958$	m1		
	$x = 1 \Rightarrow y \approx 4.5 + (2.958)(0.5)$ ... ≈ 5.98	A1F	5	ft error in y(0.5)
	Total		**5**	

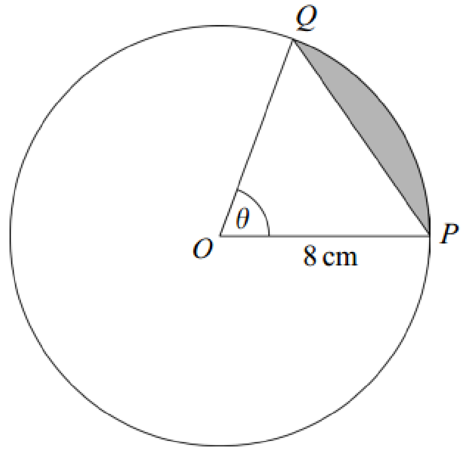
7 (a) Sketch, on the same diagram, the graphs of $y = \ln x$ and $y = \dfrac{3}{x}$ for $x > 0$. *(2 marks)*

(b) (i) Show that the equation $\ln x - \dfrac{3}{x} = 0$ has a root between $x = 2$ and $x = 3$. *(2 marks)*

(ii) With a starting value of 2.5, use the Newton-Raphson method once to find a second approximation to this root. *(4 marks)*

7 (a)	 Graph $\ln x$	B1		
	Graph $\frac{3}{x}$	B1	2	
(b)(i)	$f(3) > 0$, $f(2) < 0$ $\Rightarrow$ root in $2 < x < 3$	M1A1	2	
(ii)	$f'(x) = \frac{1}{x} + \frac{3}{x^2}$	B1		
	Use of Newton-Raphson formula	M1A1√		
	$x_1 = 2.82$	A1	4	AWRT (3 s.f) is OK
	Total		**8**	

3 The diagram shows a circle with centre O and radius 8 cm. The angle between the radii OP and OQ is θ radians.



(a) (i) Find the area of the sector OPQ in terms of θ. *(2 marks)*

(ii) Find the area of the triangle OPQ in terms of $\sin\theta$. *(2 marks)*

(iii) Hence write down the area of the shaded segment. *(1 mark)*

(b) When the area of the shaded segment is exactly one sixteenth of the area of the whole circle, θ satisfies the equation

$$8\theta - 8\sin\theta - \pi = 0.$$

(i) Show that this equation has a root between 1.3 and 1.4. *(3 marks)*

(ii) Use linear interpolation once to show that an estimate for this root is 1.37. *(3 marks)*

Q	Solution	Marks	Total	Comments
3(a)(i)	Sector area formula	M1		Allow even if formula not used
	Sector area $= 32\theta\,\text{cm}^2$	A1	2	Condone omission of units throughout
(ii)	Appropriate use of $\sin\theta$	M1		
	Triangle area $= 32\sin\theta\,\text{cm}^2$	A1	2	
(iii)	Segment area $= (32\theta - 32\sin\theta)\text{cm}^2$	A1F	1	ft c's answers, dependent on both M marks
(b)(i)	$f(1.3) \approx -0.450$, $f(1.4) \approx 0.175$	B1B1		1st B1: AWRT -0.4 or -0.5; 2nd B1: AWRT 0.2
	Sign change, so root between	E1	3	Sign change must be mentioned
(ii)	$f(1.4) - f(1.3) \approx 0.625$	B1		PI
	Considering $\frac{0.450}{0.625} (\approx 0.720)$	M1		OE
	1.37(2) convincingly found	A1	3	AG (1.37)
	Total		**11**	