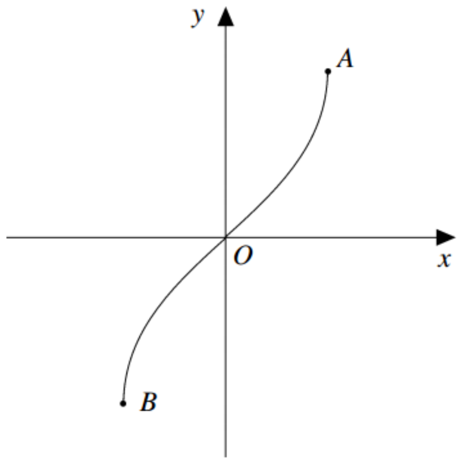
(a) The diagram shows the graph of

$$y = \sin^{-1} x.$$



Write down the coordinates of the end-points A and B. *(2 marks)*

(b) Use the mid-ordinate rule, with five strips of equal width, to estimate the value of

$$\int_0^1 \sin^{-1} x \, dx.$$

Give your answer to three decimal places. *(5 marks)*

5 (a)				**Alternative**
	Either $A\left(1, \frac{\pi}{2}\right)$ or $A(1, 90°)$	B1		x - coords ± 1 B1
	$B\left(-1, -\frac{\pi}{2}\right)$ or $B(-1, -90°)$	B1	2	y - coords $\pm \frac{\pi}{2}$ or $\pm$ 90° B1
(b)	Use of $x = 0.1, 0.3, 0.5, 0.7, 0.9$	M1		
	$y-$values: 0.1002	M1		$\sin^{-1}$(their x-values) radians
	0.3047	m1		$\sum y$ attempted (radians)
	0.5236			Accept AWRT these
	0.7754			
	1.1198			
	$I = 0.2 \times$ Sum of y-values	M1		$0.2 \times \sum$ their $y -$ values (even if degrees used)
	$= 0.565$	A1	5	CAO
	Total		**7**	

$$f(x) = 2x^2 + 3\ln(2 - x), \quad x \in \mathbb{R}, \quad x < 2.$$

(a) Show that the equation $f(x) = 0$ can be written in the form

$$x = 2 - e^{kx^2},$$

where k is a constant to be found. **(3)**

The root, α, of the equation $f(x) = 0$ is 1.9 correct to 1 decimal place.

(b) Use the iteration formula

$$x_{n+1} = 2 - e^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α to 3 decimal places and justify the accuracy of your answer. **(5)**

6.	*(a)*	$2x^2 + 3\ln(2 - x) = 0 \Rightarrow 3\ln(2 - x) = -2x^2$	
		$\ln(2 - x) = -\frac{2}{3}x^2$	M1
		$2 - x = e^{-\frac{2}{3}x^2}$	M1
		$x = 2 - e^{-\frac{2}{3}x^2}$ $\quad [\, k = -\frac{2}{3} \,]$	A1
	(b)	$x_1 = 1.90988, x_2 = 1.91212, x_3 = 1.91262, x_4 = 1.91273$	M1 A1
		$\therefore\ \alpha = 1.913$ (3dp)	A1
		$f(1.9125) = 0.0070, \ f(1.9135) = -0.020$	M1
		sign change, $f(x)$ continuous $\therefore$ root	A1
	(c)	$f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$	M1 A1
		$\therefore\ 4x - \frac{3}{2-x} = 0, \quad 4x = \frac{3}{2-x}, \quad 4x(2 - x) = 3$	M1
		$4x^2 - 8x + 3 = 0, \quad (2x - 3)(2x - 1) = 0$	M1
		$x = \frac{1}{2}, \frac{3}{2}$	A1 **(13)**

(a) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to $\int_{0.5}^{2} \frac{x}{1+x^3}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Find the exact value of $\int_{0}^{1} \frac{x^2}{1+x^3}\,dx$. *(4 marks)*

Q	Solution	Marks	Total	Comments
4(a)				
	x = 0.5, y = $\frac{4}{9}=0.\dot{4}$	B1		x values correct PI
	x = 0.75, y = $\frac{48}{91}=0.5275$	B1		At least 5 y values that would be correct to 2sf or better, or exact values. May be seen within working.
	x = 1, y = $\frac{1}{2}=0.5$			
	x = 1.25, y = $\frac{80}{189}=0.4233$			
	x = 1.5, y = $\frac{12}{35}=0.3429$			
	x = 1.75, y = $\frac{112}{407}=0.2752$			
	x = 2, y = $\frac{2}{9}=0.\dot{2}$			
	$\left[\left(\frac{4}{9}+\frac{2}{9}\right)+4\left(\frac{48}{91}+\frac{80}{189}+\frac{112}{407}\right)+2\left(\frac{1}{2}+\frac{12}{35}\right)\right]$	M1		Clear attempt to use 'their' y values within Simpson's rule
	$\int = \frac{1}{3}\times 0.25[\ \ ]$			
	$=0.605$	A1	4	Answer must be 0.605 with no extra sf (Note 0.605 with no evidence of Simpson's rule scores 0/4)
(b)	$\int_0^1 \frac{x^2}{1+x^3}dx$			
	$=\frac{1}{3}\ln\left(1+x^3\right)$	M1		$k\ln\left(1+x^3\right)$ condone missing brackets
		A1		Correct. A1 may be recovered for missing brackets if implied later
	$=\frac{1}{3}\ln(1+1)\ \left(-\frac{1}{3}\ln 1\right)$	m1		F(1) (– F(0))
	$=\frac{1}{3}\ln 2$	A1	4	ln 1 must not be left in final answer
	Alternative			
	$u=1+x^3 \quad du=3x^2dx$			
	$\int=\int\frac{du}{3u}$	(M1)		$\frac{du}{dx}$ correct and integral of form $k\int\frac{du}{u}$
	$=\frac{1}{3}[\ln u]$	(A1)		
	$=\frac{1}{3}\ln 2\ \left(-\frac{1}{3}\ln 1\right)$	(m1)		Correct substitution of correct u limits or conversion back to x and F(1) (– F(0))
	$=\frac{1}{3}\ln 2$	(A1)		ln 1 must not be left in final answer
	Total		**8**	

2 For $0 < x \leqslant 2$, the curves with equations $y = 4\ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.5. *(2 marks)*

(b) Show that the equation $4\ln x = \sqrt{x}$ can be rearranged into the form

$$x = \mathrm{e}^{\left(\frac{\sqrt{x}}{4}\right)}$$

(1 mark)

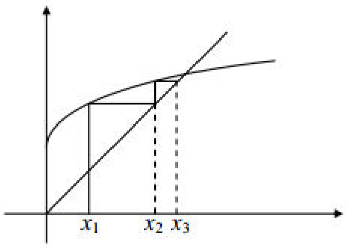
(c) Use the iterative formula

$$x_{n+1} = \mathrm{e}^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(d) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = \mathrm{e}^{\left(\frac{\sqrt{x}}{4}\right)}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x) = 4\ln x - \sqrt{x}$			Or reverse
	$\left.\begin{array}{l}\mathrm{f}(0.5) = -3.5\\ \mathrm{f}(1.5) = 0.4\end{array}\right\}$ must have both values correct	M1		Allow $\mathrm{f}(0.5) < 0$ and $\mathrm{f}(1.5) > 0$ only if f(x) defined
	Change of sign $\therefore\ 0.5 < \alpha < 1{\cdot}5$	A1	2	f(x) must be defined and all working correct, including both statement and interval (either may be written in words or symbols) **OR** comparing 2 sides: $\left.\begin{array}{ll}4\ln 0.5 = -2.8 & \sqrt{0.5} = 0.7\\ 4\ln 1.5 = 1.6 & \sqrt{1.5} = 1.2\end{array}\right\}$ (M1) At 0.5, LHS < RHS; at 1.5, LHS > RHS $\therefore 0.5 < \alpha < 1.5$ (A1)
(b)	$\ln x = \dfrac{\sqrt{x}}{4}$ or $x^4 = \mathrm{e}^{\sqrt{x}}$			Must be seen
	$x = \mathrm{e}^{\frac{\sqrt{x}}{4}}$	B1	1	AG; no errors seen
(c)	$x_2 = 1.193$	B1		
	$x_3 = 1.314$	B1	2	If B0B0 scored but either value seen correct to 2 or 4 dp, score SC1
(d)		M1		Vertical line from x_1 to curve (condone omission from x-axis to $y = x$) and then horizontal to $y = x$
		A1	2	2[nd] vertical and horizontal lines, and x_2, x_3 (not the values) must be labelled on x-axis
	Total		**7**	

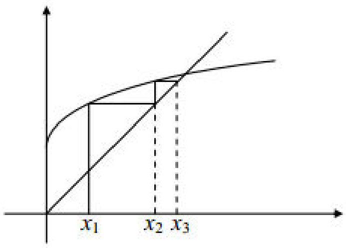
Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x) = 4\ln x - \sqrt{x}$			Or reverse
	$\left.\begin{array}{l}\mathrm{f}(0.5) = -3.5 \\ \mathrm{f}(1.5) = 0.4\end{array}\right\}$ must have both values correct	M1		Allow $\mathrm{f}(0.5) < 0$ and $\mathrm{f}(1.5) > 0$ only if f(x) defined
	Change of sign $\therefore\ 0.5 < \alpha < 1{\cdot}5$	A1	2	f(x) must be defined and all working correct, including both statement and interval (either may be written in words or symbols) **OR** comparing 2 sides: $\left.\begin{array}{ll}4\ln 0.5 = -2.8 & \sqrt{0.5} = 0.7 \\ 4\ln 1.5 = 1.6 & \sqrt{1.5} = 1.2\end{array}\right\}$ (M1) At 0.5, LHS < RHS; at 1.5, LHS > RHS $\therefore 0.5 < \alpha < 1.5$ (A1)
(b)	$\ln x = \dfrac{\sqrt{x}}{4}$ or $x^4 = \mathrm{e}^{\sqrt{x}}$			Must be seen
	$x = \mathrm{e}^{\frac{\sqrt{x}}{4}}$	B1	1	AG; no errors seen
(c)	$x_2 = 1.193$	B1		
	$x_3 = 1.314$	B1	2	If B0B0 scored but either value seen correct to 2 or 4 dp, score SC1
(d)		M1		Vertical line from x_1 to curve (condone omission from x-axis to $y = x$) and then horizontal to $y = x$
		A1	2	2nd vertical and horizontal lines, and x_2, x_3 (not the values) must be labelled on x-axis
	Total		**7**	

Figure 1