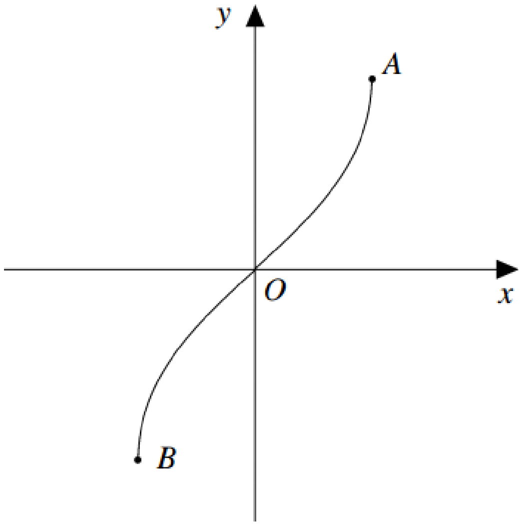
C3 Numerical methods Challenge

Challenge 1

(a) The diagram shows the graph of

$$y = \sin^{-1} x.$$



Write down the coordinates of the end-points A and B. *(2 marks)*

(b) Use the mid-ordinate rule, with five strips of equal width, to estimate the value of

$$\int_0^1 \sin^{-1} x \, dx.$$

Give your answer to three decimal places. *(5 marks)*

Challenge 2

$$f(x) = 2x^2 + 3\ln(2 - x), \quad x \in \mathbb{R}, \quad x < 2.$$

(a) Show that the equation $f(x) = 0$ can be written in the form

$$x = 2 - e^{kx^2},$$

where k is a constant to be found. **(3)**

The root, α, of the equation $f(x) = 0$ is 1.9 correct to 1 decimal place.

(b) Use the iteration formula

$$x_{n+1} = 2 - e^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α to 3 decimal places and justify the accuracy of your answer. **(5)**

Challenge 3

(a) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to $\int_{0.5}^{2} \frac{x}{1+x^3}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Find the exact value of $\int_{0}^{1} \frac{x^2}{1+x^3}\,dx$. *(4 marks)*

Final Challenge

2 For $0 < x \leqslant 2$, the curves with equations $y = 4\ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.5. *(2 marks)*

(b) Show that the equation $4\ln x = \sqrt{x}$ can be rearranged into the form

$$x = e^{\left(\frac{\sqrt{x}}{4}\right)}$$ *(1 mark)*

(c) Use the iterative formula

$$x_{n+1} = e^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(d) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = e^{\left(\frac{\sqrt{x}}{4}\right)}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*