FP1 – Numercial methods Challenge

Challenge 1

(a) Use logarithms to solve the equation $2^x = 7$, giving your answer to three significant figures. *(2 marks)*

(b) The equation

$$2^x = 7 - x$$

has a single root, α.

(i) Show that α lies between 2.0 and 2.4. *(1 mark)*

(ii) Use the bisection method to find an interval of width 0.1 in which α lies. *(3 marks)*

Challenge 2

A curve satisfies the differential equation $\frac{dy}{dx} = \sqrt{9 - x^2}$.

Starting at the point $(0, 3)$ on the curve, use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 1$, giving your answer to two decimal places. *(5 marks)*

Challenge 3

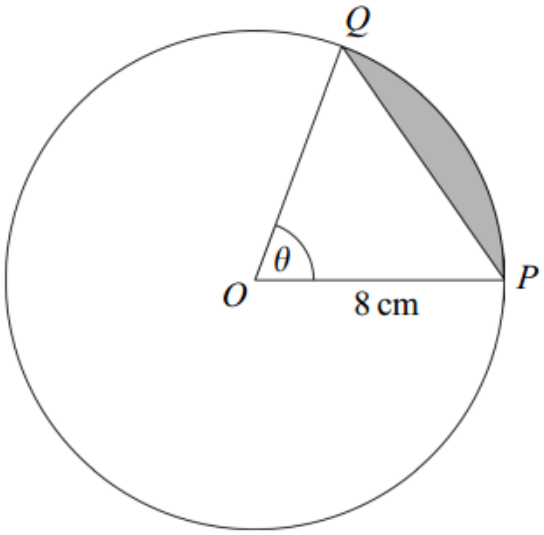
(a) Sketch, on the same diagram, the graphs of $y = \ln x$ and $y = \frac{3}{x}$ for $x > 0$. *(2 marks)*

(b) (i) Show that the equation $\ln x - \frac{3}{x} = 0$ has a root between $x = 2$ and $x = 3$. *(2 marks)*

(ii) With a starting value of 2.5, use the Newton-Raphson method once to find a second approximation to this root. *(4 marks)*

Final Challenge

The diagram shows a circle with centre O and radius 8 cm. The angle between the radii OP and OQ is θ radians.



(a) (i) Find the area of the sector OPQ in terms of θ. *(2 marks)*

(ii) Find the area of the triangle OPQ in terms of $\sin\theta$. *(2 marks)*

(iii) Hence write down the area of the shaded segment. *(1 mark)*

(b) When the area of the shaded segment is exactly one sixteenth of the area of the whole circle, θ satisfies the equation

$$8\theta - 8\sin\theta - \pi = 0.$$

(i) Show that this equation has a root between 1.3 and 1.4. *(3 marks)*

(ii) Use linear interpolation once to show that an estimate for this root is 1.37. *(3 marks)*