Polynomial Functions

The cubic polynomial $x^3 + ax^2 + bx + 4$, where a and b are constants, has factors $x - 2$ and $x + 1$. Use the factor theorem to find the values of a and b. *(6 marks)*

4	$8+4a+2b+4$ $=0$ $-1+a-b+4=0$ $a=-3,\quad b=0$	M1 A1 M1A1 A1A1	6	a.e.f.
	or s.r. maximum mark $5/6$: $(x\quad)(x-2)(x+1)$ $(x-2)(x-2)(x+1)$ Multiplying out $a=-3,\quad b=0$	 M1 A1 M1 A1A1		
	Total		**6**	

Given that $f(x) = x^3 - 4x^2 - x + 4$,

(a) find f(1) and f(2), *(2 marks)*

(b) factorise f(x) into the product of three linear factors. *(3 marks)*

Q	Solution	Marks	Total	Comments
1 (a)	0, −6	B1B1	2	
(b)	$(x-1)(x^2-3x-4)$ $(x-1)(x+1)(x-4)$	B1 M1A1	 3	for $x-1$ factor allow separate factors $(x^2-1)(x-4)$ SR1
	Total		**5**	

The polynomial $f(x)$ is given by

$$f(x) = x^3 + px^2 + x + 54,$$

where p is a real number. When $f(x)$ is divided by $x + 3$, the remainder is -3.

Use the Remainder Theorem to find the value of p. *(3 marks)*

Q	Solution	Marks	Total	Comments
1	Substitute $x=\pm 3$ $x=-3$ correctly substituted $p=-3$	M1 A1 A1F	 3	 Division earns 0 marks
	Total		**3**	

$$f(x) = 6x^3 + ax^2 + bx - 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$ there is no remainder.

When $f(x)$ is divided by $(2x - 1)$ the remainder is -15

(a) Find the value of a and the value of b.

(5)

(b) Factorise $f(x)$ completely.

(4)

Question Number		Scheme	Marks
6	(a)	$f(2) = 16 + 40 + 2a + b$ **or** $f(-1) = 1 - 5 - a + b$	M1 A1
		Finds 2nd remainder and equates to 1st $\Rightarrow$ $16 + 40 + 2a + b = 1 - 5 - a + b$	M1 A1
		$a = -20$	A1cso (5)
	(b)	$f(-3) = (-3)^4 + 5(-3)^3 - 3a + b = 0$	M1 A1ft
		$81 - 135 + 60 + b = 0$ gives $b = -6$	A1 cso
			(3) **[8]**