7 A particle is projected from a horizontal surface at a speed V and at an angle α above the horizontal.

(a) Prove that the maximum height of the particle is $\dfrac{V^2 \sin^2 \alpha}{2g}$. *(6 marks)*

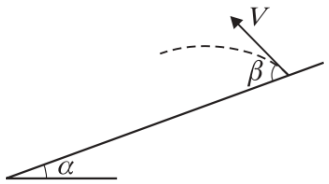
(b) A ball is hit from ground level. The ball initially moves at an angle of 60° above the horizontal. The maximum height of the ball is 6 metres above the ground. Modelling the ball as a particle:

(i) find the initial speed of the ball; *(3 marks)*

(ii) find the range of the ball. *(4 marks)*

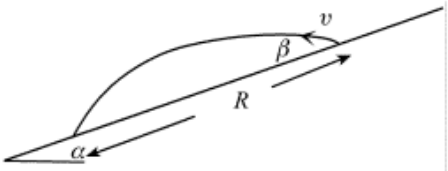
Q	Solution	Marks	Total	Comments
7(a)	$0 = V\sin\alpha - gt$	M1		Equation for t at max. height
	$t = \dfrac{V\sin\alpha}{g}$	M1 A1		Solving for t Correct t
	$H = V\sin\alpha \times \dfrac{V\sin\alpha}{g} - \dfrac{1}{2} \times g \times \left(\dfrac{V\sin\alpha}{g}\right)^2$	M1 A1		Substitution into equation for height Correct expression
	$= \dfrac{V^2 \sin^2 \alpha}{2g}$	A1	6	Correct final result from correct working
(b)	$6 = \dfrac{V^2}{2g} \times \dfrac{3}{4}$	M1		Substitution into given formula
(i)	$V = \sqrt{16g} = 12.5\ \text{ms}^{-1}$	M1 A1	3	Solving for V Correct value
(ii)	$t = \dfrac{2\sqrt{16g}\ \sin 60^\circ}{g} = 2.213$	M1 A1		Finding time of flight Correct time of flight
	$R = \sqrt{16g}\cos 60^\circ \times 2.213 = 13.9\ \text{m}$	M1 A1	4	Finding range Correct range
	Total		**13**	

4 A ball is thrown with velocity V **down** a plane which is inclined at an angle α to the horizontal.



If β is the angle which the initial velocity of the ball makes with the inclined plane, show that the range down the plane along the line of greatest slope is

$$\frac{2V^2}{g\cos^2\alpha}\sin\beta\cos(\alpha-\beta).$$ *(8 marks)*

Question Number and part	Solution	Marks	Total Marks
4	Consider particle as shown  Consider particle fired at β to plane Motion perpendicular to plane $s = v\sin\beta t - \frac{1}{2}g\cos\alpha\, t^2$ Hits plane when $s = 0$, $\therefore t = \frac{2v\sin\beta}{g\cos\alpha}$ $(t \neq 0)$ Motion along plane; $R = v\cos\beta t + \frac{1}{2}g\sin\alpha\, t^2$ $= v\cos\beta.\frac{2v\sin\beta}{g\cos\alpha} + \frac{1}{2}g\sin\alpha.\frac{4v^2\sin^2\beta}{g^2\cos^2\alpha}$ $= \frac{2v^2\sin\beta(\cos\beta\cos\alpha + \sin\alpha\sin\beta)}{g\cos^2\alpha}$ $= \frac{2v^2}{g\cos^2\alpha}\sin\beta\cos(\alpha-\beta)$ **or** Horizontal distance is $\frac{2v\sin\beta}{g\cos\alpha} \times v\cos(\alpha-\beta)$ $\therefore$ distance along plane is $\frac{2v^2}{g\cos^2\alpha}\sin\beta\cos(\alpha-\beta)$	M1 A1 A1 M1 A1 M1 A1 A1 M2 A1 M1 A1	8
	Total		**8**

6 A slope is inclined at an angle of 20° below the horizontal. A ball is projected at a speed of 30 m s^{-1} from the slope at an angle of 40° above the slope. The ball moves in a plane that contains the line of greatest slope of the plane.

(a) Find the time of flight of the ball, given that it moves down the slope. *(5 marks)*

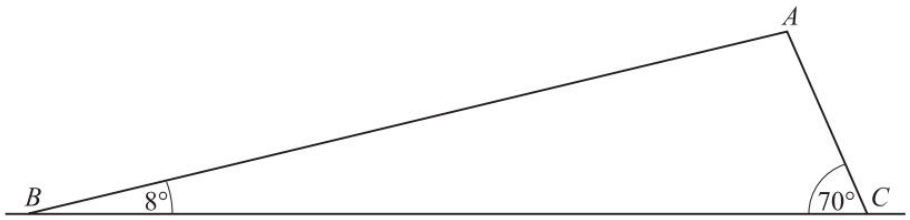
(b) Find the range of the ball. *(4 marks)*

(c) Find the speed of the ball when it hits the slope, giving your answer correct to 2 significant figures. *(4 marks)*

Question	Solution	Marks	Total
6(a)	$y = 30\sin 40°t - 4.9\cos 20°t^2$	M1A1	
	$0 = 30\sin 40°t - 4.9\cos 20°t^2$	A1	
	$t = 0$ or $t = \dfrac{30\sin 40°}{4.9\cos 20°} = 4.188\,\text{s}$	M1 A1	5
(b)	$x = 30\cos 40° \times 4.188 + 4.9\sin 20° \times 4.188^2$	M1A1	
	$= 126\,\text{m}$	A1	
		A1	4
(c)	$v_x = 30\cos 40° + 9.8\sin 20° \times 4.188$		
	$= 37.02$	M1A1	
	$v_y = 30\sin 40° - 9.8\cos 20° \times 4.188$	A1	
	$= -19.28$		
	$v = \sqrt{37.02^2 + (-19.28)^2} = 42\,\text{m s}^{-1}$	A1	4
	Total		**13**

5 A large sand dune can be modelled as a triangular prism with two inclined planes, one plane, AB, inclined at 8° to the horizontal and the other plane, AC, inclined at 70° to the horizontal.

A is a point on the top ridge of the sand dune, as shown in the diagram.



A football is kicked up a line of greatest slope on that part of the sand dune which is inclined at 8° to the horizontal. When it reaches the top ridge the football is travelling with velocity u, and it then moves freely under gravity until it strikes the inclined plane AC.

Find, in terms of u:

(a) the time for which the football is not in contact with the sand; *(6 marks)*

(b) the distance down the slope from A at which the football strikes the sand dune AC. *(6 marks)*

Question Number and part	Solution	Marks	Total Marks	Comments
5(a)	Considering initial velocity on incline plane, the ball's velocity			
	perpendicular to plane is $u\cos 12$ down plane is $u\sin 12$	B1		
	Consider motion perpendicular to plane; acceleration is $g\cos 70$	M1 A1		Needs inclined plane
	$0 = u\cos 12t - \frac{1}{2}g\cos 70t^2$	M1		Using $s = ut + \frac{1}{2}at^2$
	$t = 0$ or $\dfrac{2u\cos 12}{g\cos 70}$	M1 A1	6	
	$\therefore$ Time is $\dfrac{2u\cos 12}{g\cos 70} = 0.584u$			
(b)	Consider motion down plane; acceleration is $g\sin 70$	M1 A1		
	$s = u\sin 12t + \frac{1}{2}\cdot g\sin 70t^2$	M1 A1		Using $s = ut + \frac{1}{2}at^2$
	$= 2u^2\dfrac{\sin 12\cos 12}{g\cos 70} + 2\dfrac{u^2\cos^2 12\sin 70}{g\cos^2 70}$	M1		
	Distance $= 1.69u^2$	A1	6	
	Total		**12**	