Quadratic Equations

(a) (i) Express $x^2 + 12x + 11$ in the form $(x + a)^2 + b$, finding the values of a and b. *(2 marks)*

(ii) State the minimum value of the expression $x^2 + 12x + 11$. *(1 mark)*

(b) Determine the values of k for which the quadratic equation

$$x^2 + 3(k - 2)x + (k + 5) = 0$$

has equal roots. *(4 marks)*

Question	Solution	Marks	Total Marks	Comments
5 (a)(i)	$(x+6)^2 + 11 - 36$	B1		or equivalent
	$b = -25$	B1	**(2)**	
(ii)	Minimum value of b (follow through) $\underline{-25}$	B1 √	**(1)**	
(b)	$9(k-2)^2 - 4(k+5)$	M1		Use of $b^2 - 4ac$
	$9k^2 - 40k + 16 = 0$	A1		
	$(k-4)(9k-4)$ or formula	M1		factors or good attempt at quadratic
	$\underline{k = 4 \ , \ \frac{4}{9}}$	A1	**(4)**	
		TOTAL	**7**	

The quadratic equation

$$x^2 + (3 - k)x + 5 - k^2 = 0$$

is to be considered for different values of the constant k.

(a) When $k = 7$:

(i) show that $x^2 - 4x - 44 = 0$; *(1 mark)*

(ii) find the roots of this equation, giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. *(2 marks)*

(b) When the quadratic equation $x^2 + (3 - k)x + 5 - k^2 = 0$ has equal roots:

(i) show that $5k^2 - 6k - 11 = 0$; *(3 marks)*

(ii) hence find the possible values of k. *(2 marks)*

5(a)(i)	$x^2+(3-7)x+5-49=0$ $\Rightarrow x^2-4x-44=0$	B1	1	Be convinced - no missing brackets etc **ag** Must have = 0
(ii)	Use of quadratic equation formula or attempt to complete square	M1		Condone one slip $\frac{4\pm\sqrt{192}}{2}$
	$\Rightarrow (x=)\ 2\pm4\sqrt{3}$	A1	2	
(b)(i)	Discriminant b^2-4ac	M1		Used - must involve k
	$(3-k)^2-4(5-k^2)$	A1		$9-6k+k^2-20+4k^2$
	$\Rightarrow 5k^2-6k-11=0$	A1	3	**ag** must use " = 0 " condition
(ii)	$(5k-11)(k+1)=0$	M1		Attempt to solve or factorise
	$\Rightarrow k=-1,\ \frac{11}{5}$	A1	2	
	Total		**8**	

(a) (i) Express $x^2+8x+11$ in the form $(x+p)^2+q$. *(2 marks)*

(ii) Hence, or otherwise, find the coordinates of the minimum point of the curve with equation $y=x^2+8x+11$. *(2 marks)*

(b) Describe in detail the geometrical transformation which maps the graph of $y=x^2$ onto the graph of $y=x^2+8x+11$. *(3 marks)*

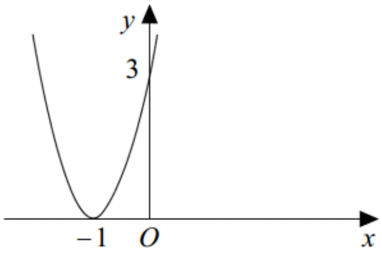
(c) Determine the condition on k for which the equation

$$x^2+8x+11-k=0$$

has no real solutions. *(3 marks)*

Question Number and part	Solution	Marks	Total marks	Comments
6(a)(i)	$(x+4)^2-5$	B1 B1	2	$p=4;\quad q=-5$
(ii)	Minimum $(-4,-5)$ **or** $x=-4,\quad y=-5$	B1√ B1√	2	ft their p and q (or correct)
(b)	Translation through $\begin{bmatrix}-4\\-5\end{bmatrix}$	M1 A1 A1	3	M1 for "shift" if one term correct or $\begin{bmatrix}*\\*\end{bmatrix}$ if one term correct , etc
(c)	No real roots when $(b^2-4ac)<0$	B1		May be stated and not used
	$64-4(11-k)$	M1		Condone sign error with k (or one slip) May be part of quadratic equation formula
	$k<-5$	A1	3	cso
	Total		**10**	

The graph of $y = 3(x + 1)^2$ is sketched below.



(a) Describe fully a sequence of geometrical transformations that would map the graph of $y = x^2$ onto the graph of $y = 3(x + 1)^2$. *(4 marks)*

(b) (i) Express $3(x + 1)^2$ in the form $px^2 + qx + r$. *(1 mark)*

(ii) Find the gradient of the curve with equation $y = 3(x + 1)^2$ at the point where $x = 4$. *(3 marks)*

(c) (i) Show that the curve with equation $y = 3(x + 1)^2$ and the line with equation $y = kx - 9$ intersect when

$$3x^2 + (6 - k)x + 12 = 0$$ *(1 mark)*

(ii) Find the values of k for which the quadratic equation

$$3x^2 + (6 - k)x + 12 = 0$$

has equal roots. *(4 marks)*

(iii) State the geometrical relationship between the line $y = kx - 9$ and the curve $y = 3(x + 1)^2$ for these values of k. *(1 mark)*

Question	Solution	Marks	Total	Comments
8(a)	One-way stretch in y-direction	M1		Allow method mark only for description such as move / shift / enlarge if other feature such as scaling factor is correct
	Scale factor 3	A1		
	Translation in x-direction	M1		
	Vector $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	A1	4	
(b)(i)	$3x^2 + 6x + 3$	B1	1	
(ii)	$\frac{dy}{dx} = 6x + 6$	M1		Attempt at differentiation
		A1√		ft from (i)
	When $x = 4$, gradient $= 30$	A1√	3	
(c)(i)	$3x^2 + 6x + 3 = kx - 9$			
	$\Rightarrow 3x^2 + (6 - k)x + 12 = 0$	B1	1	ag
(ii)	$(6 - k)^2 = 144$	B1		Discriminant $= 0$
	$6 - k = \pm 12$	M1		Attempt to solve for k
	$k = -6$	A1		
	$k = 18$	A1	4	
(iii)	Line is a tangent to the curve	E1	1	Single point of intersection etc
	Total		**14**	