2 The quadratic equation

$$x^2 + px + 2 = 0$$

has roots α and β.

(a) Write down the value of $\alpha\beta$. *(1 mark)*

(b) Express in terms of p:

(i) $\alpha + \beta$; *(1 mark)*

(ii) $\alpha^2 + \beta^2$. *(2 marks)*

(c) Given that $\alpha^2 + \beta^2 = 5$, find the possible values of p. *(1 mark)*

2(a)	$\alpha\beta = 2$	B1	1	if seen anywhere
(b)(i)	$\alpha + \beta = -p$	B1	1	if seen anywhere
(ii)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	M1		correct use of $(\alpha+\beta)^2 - 2\alpha\beta$
	$= p^2 - 4$	A1F	2	ft from their $(\alpha+\beta)$ and $\alpha\beta$
(c)	$p^2 - 4 = 5 \Rightarrow p = \pm 3$	A1F	1	No ft from $\alpha^2 + \beta^2 = (\alpha+\beta)^2$
	Total		**5**	

1 (a) The quadratic equation $2x^2 - 6x + 1 = 0$ has roots α and β.

Write down the numerical values of:

(i) $\alpha\beta$; *(1 mark)*

(ii) $\alpha + \beta$. *(1 mark)*

(b) Another quadratic equation has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Find the numerical values of:

(i) $\frac{1}{\alpha} \times \frac{1}{\beta}$; *(1 mark)*

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, writing your answer in the form $x^2 + px + q = 0$. *(2 marks)*

Q	Solution	Marks	Total	Comments
1 (a)(i)	$\alpha\beta = \frac{1}{2}$	B1		
(ii)	$\alpha + \beta = 3$	B1	2	
(b)(i)	$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = 2$	B1√	1	
(ii)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = 6$	M1A1√	2	
(c)	$x^2 - (sum)x + (product) = 0$ $x^2 - 6x + 2 = 0$	M1 A1√	2	Replace x by $\frac{1}{x}$ $2\left(\frac{1}{x}\right)^2 - 6\left(\frac{1}{x}\right) + 1 = 0$ $\frac{2}{x^2} - \frac{6}{x} + 1 = 0$ × by x^2 to give $x^2 - 6x + 2 = 0$
	Total		**7**	

1 (a) The roots of the quadratic equation $x^2 + 4x - 3 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$;

(ii) $\left(\alpha^2 + \frac{2}{\beta}\right)\left(\beta^2 + \frac{2}{\alpha}\right)$. *(6 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots

$$\left(\alpha^2 + \frac{2}{\beta}\right) \text{ and } \left(\beta^2 + \frac{2}{\alpha}\right).$$ *(4 marks)*

Question Number and part	Solution	Marks	Total marks	Comments
1(a)(i)	$\alpha + \beta = -4; \quad \alpha\beta = -3$	B1		Likely to be earned in (ii)
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		oe
	$= 16 + 6 = 22$	A1		
(ii)	$\alpha^2\beta^2 + 2(\alpha + \beta) + \frac{4}{\alpha\beta}$	B1		
	$9 - 8 - \frac{4}{3}$	M1		Substitution into similar form as above
	$= -\frac{1}{3}$	A1	6	
(b)	Sum of roots $= \alpha^2 + \beta^2 + \frac{2}{\alpha} + \frac{2}{\beta}$			
	$= \alpha^2 + \beta^2 + \frac{2}{\alpha\beta}(\alpha + \beta)$	M1		
	$= 22 + \frac{2}{-3} \times -4 \quad = \frac{74}{3}$	A1		
	New equation $y^2 - (\text{sum of new roots})\, y + \text{product} = 0$	M1		
	$\Rightarrow y^2 - \frac{74}{3}y - \frac{1}{3} = 0$			
	$\Rightarrow 3y^2 - 74y - 1 = 0$	A1ft	4	(ft any variable fractional values) Must have = 0
	Total		**10**	

9 The roots of the quadratic equation $x^2 - 3x + 1 = 0$ are α and β.

(a) Without solving the equation:

(i) show that $\alpha^2 + \beta^2 = 7$; *(3 marks)*

(ii) find the value of $\alpha^3 + \beta^3$. *(3 marks)*

(b) (i) Show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$. *(1 mark)*

(ii) Hence find the value of $\alpha^4 + \beta^4$. *(2 marks)*

(c) Determine a quadratic equation with integer coefficients which has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$. *(5 marks)*

Question Number and part	**Solution**	**Marks**	**Total**	**Comments**
9(a)(i)	$\alpha + \beta = 3$; $\quad \alpha\beta = 1$	B1		Withhold if obviously incorrect in (ii)
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		
	$= 9 - 2 = 7$	A1	3	**ag** However, condone $(-3)^2$
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$	M1 A1		Good attempt at any equivalent Correct formula
	$= 18$	A1	3	
(b)(i)	$(\alpha^2 + \beta^2)^2 = \alpha^4 + 2\alpha^2\beta^2 + \beta^4$			
	$\Rightarrow \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$	B1	1	**ag** Be generous here.
(ii)	$\alpha^4 + \beta^4 = 49 - 2$	M1		Substitute candidate's $\alpha\beta$
	$= 47$	A1	2	
(c)	Sum of roots $= \alpha^3 + \beta^3 - (\alpha + \beta)$	M1		
	$= 15$	A1		
	Product $= (\alpha\beta)^3 + \alpha\beta - (\alpha^4 + \beta^4)$	M1		Condone one slip
	$= 1 + 1 - 47 = -45$	A1		
	New equation			
	$y^2 - 15y - 45 = 0$	B1√	5	ft any variable, integer coefficients Must have = 0
	Total		**14**	