FP1 – Quadratic roots Challenge

Challenge 1

The quadratic equation

$$x^2 + px + 2 = 0$$

has roots α and β.

(a) Write down the value of $\alpha\beta$. *(1 mark)*

(b) Express in terms of p:

(i) $\alpha + \beta$; *(1 mark)*

(ii) $\alpha^2 + \beta^2$. *(2 marks)*

(c) Given that $\alpha^2 + \beta^2 = 5$, find the possible values of p. *(1 mark)*

Challenge 2

(a) The quadratic equation $2x^2 - 6x + 1 = 0$ has roots α and β.

Write down the numerical values of:

(i) $\alpha\beta$; *(1 mark)*

(ii) $\alpha + \beta$. *(1 mark)*

(b) Another quadratic equation has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Find the numerical values of:

(i) $\frac{1}{\alpha} \times \frac{1}{\beta}$; *(1 mark)*

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, writing your answer in the form $x^2 + px + q = 0$. *(2 marks)*

Challenge 3

(a) The roots of the quadratic equation $x^2 + 4x - 3 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$;

(ii) $\left(\alpha^2 + \frac{2}{\beta}\right)\left(\beta^2 + \frac{2}{\alpha}\right)$. *(6 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots

$$\left(\alpha^2 + \frac{2}{\beta}\right) \text{ and } \left(\beta^2 + \frac{2}{\alpha}\right).$$ *(4 marks)*

Final Challenge

The roots of the quadratic equation $x^2 - 3x + 1 = 0$ are α and β.

(a) Without solving the equation:

(i) show that $\alpha^2 + \beta^2 = 7$; *(3 marks)*

(ii) find the value of $\alpha^3 + \beta^3$. *(3 marks)*

(b) (i) Show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$. *(1 mark)*

(ii) Hence find the value of $\alpha^4 + \beta^4$. *(2 marks)*

(c) Determine a quadratic equation with integer coefficients which has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$. *(5 marks)*