2 Axes Ox, Oy and Oz are defined respectively in the north, west and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions.

At 3 pm, an aeroplane, A, is 1.7 miles high above a radar beacon, R.

At 2 pm, a weather balloon, B, was released from a point Q with position vector $(20\mathbf{i} + 5\mathbf{j} + 0.1\mathbf{k})$ relative to R.

The units of distance are miles.

The weather balloon has a constant velocity $(10\mathbf{i} + 15\mathbf{j} + 3\mathbf{k})$ miles per hour.

(a) Find the position vector of B relative to R at 3 pm. *(2 marks)*

At 3 pm, the velocity of A is $(280\mathbf{i} + 265\mathbf{j} + 10\mathbf{k})$ miles per hour.

Assume that the velocity of the plane is constant for the next 30 minutes.

(b) Find the velocity of B relative to A during these 30 minutes. *(1 mark)*

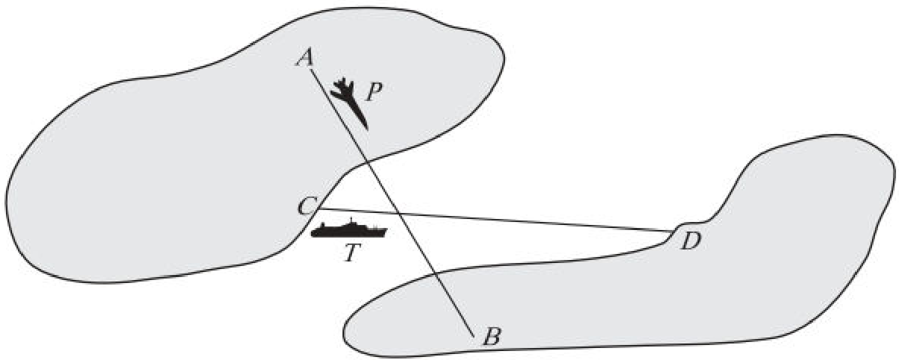
(c) Find the distance, in miles, between the aeroplane and the weather balloon at 3.30 pm. *(2 marks)*

2 (a)	B relative to R at 3 pm is			
	$\begin{pmatrix}20\\5\\0.1\end{pmatrix}+\begin{pmatrix}10\\15\\3\end{pmatrix}$	M1		
	$=\begin{pmatrix}30\\20\\3.1\end{pmatrix}$	A1	**(2)**	
(b)	Velocity of B relative to A is $\mathbf{v}_B-\mathbf{v}_A$			
	$=\begin{pmatrix}10\\15\\3\end{pmatrix}-\begin{pmatrix}280\\265\\10\end{pmatrix}$			
	$=\begin{pmatrix}-270\\-250\\-7\end{pmatrix}$	B1	**(1)**	
(c)	Position of B relative to A is			
	$\begin{pmatrix}30\\20\\1.4\end{pmatrix}+\begin{pmatrix}-270\\-250\\-7\end{pmatrix}\times 0.5=\begin{pmatrix}-105\\-105\\-2.1\end{pmatrix}$	M1		
	Distance $=\sqrt{105^2+105^2+2.1^2}=149$ miles	A1	**(2)**	
		TOTAL	**(5)**	

3 Axes Ox, Oy and Oz are defined respectively in the East, North and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions respectively. The units of distance are metres and the units of velocity are metres per second.

A small plane, P, is flying between two airports, A and B, on the two islands shown.

A boat, T, is travelling between two harbours, C and D, on the two islands.



At 10 am, the plane leaves A and the boat leaves C. Harbour C has position vector $80\mathbf{i} - 6000\mathbf{j}$ relative to A.

After take-off, the plane travels with constant velocity $30\mathbf{i} - 25\mathbf{j} + 2.1\mathbf{k}$. After leaving harbour, the boat has a constant velocity $18\mathbf{i} - \mathbf{j}$. Time t is measured in seconds after 10 am.

(a) State the position of T relative to P at 10 am. *(1 mark)*

(b) Find the velocity of T relative to P. *(2 marks)*

(c) Find an expression for the distance, S metres, which the plane and the boat are apart at time t. You do **not** need to simplify your expression. *(4 marks)*

(d) Find t when S^2 is a minimum. Hence state the time at which the plane and the boat are nearest to each other. *(4 marks)*

(e) Show that at 10.04 am the distance between the plane and the boat is less than 3 km. *(3 marks)*

3 (a)	$\mathbf{r}_{T\text{ rel }P} = \mathbf{r}_T - \mathbf{r}_P$ $= 80\mathbf{i} - 6000\mathbf{j}$	B1	1	
(b)	$\mathbf{v}_{T\text{ rel }P} = \mathbf{v}_T - \mathbf{v}_P$ $= \begin{pmatrix} 18 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 30 \\ -25 \\ 2.1 \end{pmatrix}$ $= \begin{pmatrix} -12 \\ 24 \\ -2.1 \end{pmatrix}$	M1 A1	 2	
(c)	$\mathbf{r}_{T\text{ rel }P} = \begin{pmatrix} 80-12t \\ -6000+24t \\ -2.1t \end{pmatrix}$ $D = \left\{(80-12t)^2 + (-6000+24t)^2 + (2.1t)^2\right\}^{\frac{1}{2}}$	M1 A1 M1 A1	 4	
(d)	$\dfrac{\mathrm{d}D^2}{\mathrm{d}t} = -24(80-12t) + 48(-6000+24t) + 8.82t$ $\dfrac{\mathrm{d}D^2}{\mathrm{d}t} = 0 \quad \Rightarrow$ $-1920 + 288t - 288000 + 1152t + 8.82t = 0$ $1448.82t = 289920$ $t = 200.10$ $\therefore$ time is 10.03 and 20 sec	M1 M1 A1 A1	 4	
(e)	When $t = 240$ $D = \left\{2800^2 + 240^2 + 504^2\right\}^{\frac{1}{2}}$ $= 2855\text{m}$ $< 3\text{km}$	B1 M1 A1	 3	
	Total		**14**	

5 Axes Ox, Oy and Oz are defined respectively in the East, North and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions. The units of distance are metres and the units of velocity are metres per minute.

At 8 am, a hot air balloon, B, is 120 metres above a rock, R, situated on level ground in a wildlife national park. A tourist in the hot air balloon sees a lion, L, in the distance at a point A, which has position vector $200\mathbf{i} - 60\mathbf{j}$ relative to R.

The lion is walking with constant velocity $4\mathbf{i} + 8\mathbf{j}$.

The balloon has a constant velocity of $15\mathbf{i} + 6\mathbf{j} - 3.2\mathbf{k}$.

(a) Find the position of L relative to B at 8 am. *(2 marks)*

(b) Assume that the velocity of the lion and the balloon are constant for the next 25 minutes. Time t is measured in minutes after 8 am.

(i) Find the velocity of L relative to B during these 25 minutes. *(2 marks)*

(ii) Find an expression for the distance, in metres, which the lion and the hot air balloon are apart at time t, where $0 < t < 25$. You do **not** need to simplify your expression. *(2 marks)*

(iii) Hence find the time at which the lion and the balloon are nearest to each other. *(4 marks)*

5(a)	$r_L - r_B$ $=\begin{pmatrix}200\\-60\\0\end{pmatrix}-\begin{pmatrix}0\\0\\120\end{pmatrix}$ $=\begin{pmatrix}200\\-60\\-120\end{pmatrix}$	M1 A1	2	position of L relative to B M1 only for $\begin{pmatrix}200\\-60\end{pmatrix}$
(b)(i)	$v_L - v_B$ $=\begin{pmatrix}4\\8\\0\end{pmatrix}-\begin{pmatrix}15\\6\\-3.2\end{pmatrix}$ $=\begin{pmatrix}-11\\2\\3.2\end{pmatrix}$	M1 A1	2	velocity of L relative to B
(ii)	**Distance (at time t) is** $\begin{pmatrix}200\\-60\\-120\end{pmatrix}+t\begin{pmatrix}-11\\2\\3.2\end{pmatrix}$ $\therefore D=\left\{(200-11t)^2+(-60+2t)^2+(-120+3.2t)^2\right\}^{\frac{1}{2}}$ A1	M1	2	
(iii)	$\frac{dD^2}{dt}=-22(200-11t)+4(-60+2t)+6.4(-120+3.2t)$ M1A1 $\frac{dD^2}{dt}=0\Rightarrow$ $-4400+242t-240+8t-768+20.48t=0$ $270.48t=5408$ $t=19.99$ Time is 8.20am	M1 A1	4	20 min etc M1 A1 M1 only
	Total		**10**	

7 The unit vectors **i** and **j** are defined in the east and north directions respectively. The unit of distance is kilometres and the unit of velocity is kilometres per hour.

Initially, two ships P and Q are 2 kilometres apart with P due south of Q.

Ship Q is travelling with velocity $10\sqrt{3}\mathbf{i} - 10\mathbf{j}$ kilometres per hour.

The maximum speed of ship P is 8 kilometres per hour.

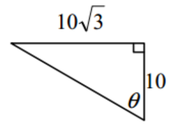
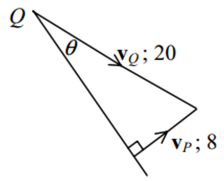
(a) Find the speed of ship Q, and the bearing on which it is travelling. *(3 marks)*

(b) Ship P travels to ensure that it approaches Q as closely as possible.

(i) Find the direction in which P travels. *(4 marks)*

(ii) Show that the velocity of Q relative to P is $11\mathbf{i} - 15\mathbf{j}$ correct to 2 significant figures. *(3 marks)*

(iii) Find the shortest distance between the ships. *(4 marks)*

7(a)	Speed of Q is 20 km/h	B1		
				
	$\tan\theta = \frac{10\sqrt{3}}{10}$	M1		
	Bearing is 120°	A1	3	
b(i)	Ship P will travel so that v_P is perpendicular to the relative velocity	M1		(If not gained, can gain M1 in (ii) and all marks in (iii))
				
	$\sin\theta = \frac{8}{20} = 0.4$	m1		Dependent on M1 above
	$\theta = 23.6°$	A1		
	Bearing of ship P is 054°	B1	4	Dependent on first M1 Accept 053.6°
(ii)	Velocity of P is $8 \sin 53.6\mathbf{i} + 8 \cos 53.6\mathbf{j}$	B1		Dependent on M1,M1 in (i)
	Velocity of Q relative to P is $v_Q - v_P$			
	$= (10\sqrt{3}\,\mathbf{i} - 10\mathbf{j}) - (6.439\mathbf{i} + 4.7498\mathbf{j})$	M1		
	$= 10.88\mathbf{i} - 14.75\mathbf{j}$			
	$= 11\mathbf{i} - 15\mathbf{j}$ [to 2 significant figures]	A1	3	