M3 Relative velocity Challenge

Challenge 1

Axes Ox, Oy and Oz are defined respectively in the north, west and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions.

At 3 pm, an aeroplane, A, is 1.7 miles high above a radar beacon, R.

At 2 pm, a weather balloon, B, was released from a point Q with position vector $(20\mathbf{i} + 5\mathbf{j} + 0.1\mathbf{k})$ relative to R.

The units of distance are miles.

The weather balloon has a constant velocity $(10\mathbf{i} + 15\mathbf{j} + 3\mathbf{k})$ miles per hour.

(a) Find the position vector of B relative to R at 3 pm. *(2 marks)*

At 3 pm, the velocity of A is $(280\mathbf{i} + 265\mathbf{j} + 10\mathbf{k})$ miles per hour.

Assume that the velocity of the plane is constant for the next 30 minutes.

(b) Find the velocity of B relative to A during these 30 minutes. *(1 mark)*

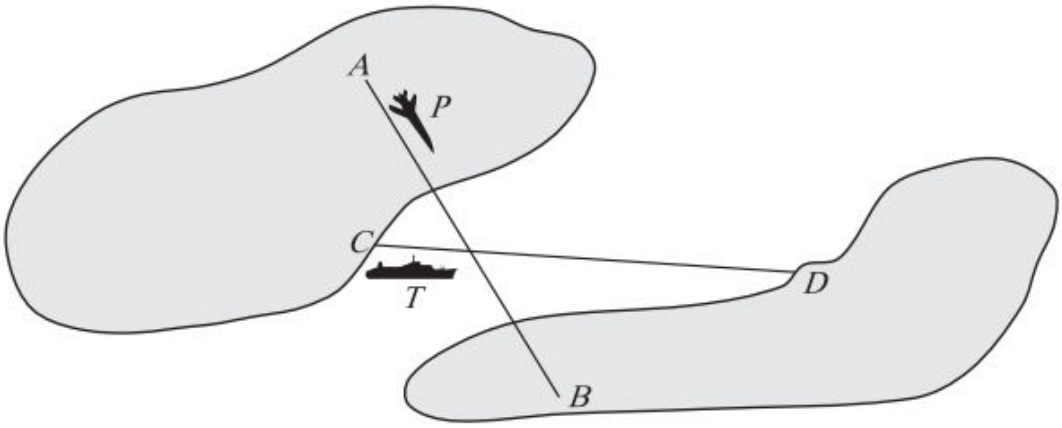
(c) Find the distance, in miles, between the aeroplane and the weather balloon at 3.30 pm. *(2 marks)*

Challenge 2

Axes Ox, Oy and Oz are defined respectively in the East, North and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions respectively. The units of distance are metres and the units of velocity are metres per second.

A small plane, P, is flying between two airports, A and B, on the two islands shown.

A boat, T, is travelling between two harbours, C and D, on the two islands.



At 10 am, the plane leaves A and the boat leaves C. Harbour C has position vector $80\mathbf{i} - 6000\mathbf{j}$ relative to A.

After take-off, the plane travels with constant velocity $30\mathbf{i} - 25\mathbf{j} + 2.1\mathbf{k}$. After leaving harbour, the boat has a constant velocity $18\mathbf{i} - \mathbf{j}$. Time t is measured in seconds after 10 am.

(a) State the position of T relative to P at 10 am. *(1 mark)*

(b) Find the velocity of T relative to P. *(2 marks)*

(c) Find an expression for the distance, S metres, which the plane and the boat are apart at time t. You do **not** need to simplify your expression. *(4 marks)*

(d) Find t when S^2 is a minimum. Hence state the time at which the plane and the boat are nearest to each other. *(4 marks)*

(e) Show that at 10.04 am the distance between the plane and the boat is less than 3 km. *(3 marks)*

Challenge 3

Axes Ox, Oy and Oz are defined respectively in the East, North and vertically upwards directions. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined in the x, y and z directions. The units of distance are metres and the units of velocity are metres per minute.

At 8 am, a hot air balloon, B, is 120 metres above a rock, R, situated on level ground in a wildlife national park. A tourist in the hot air balloon sees a lion, L, in the distance at a point A, which has position vector $200\mathbf{i} - 60\mathbf{j}$ relative to R.

The lion is walking with constant velocity $4\mathbf{i} + 8\mathbf{j}$.

The balloon has a constant velocity of $15\mathbf{i} + 6\mathbf{j} - 3.2\mathbf{k}$.

(a) Find the position of L relative to B at 8 am. *(2 marks)*

(b) Assume that the velocity of the lion and the balloon are constant for the next 25 minutes. Time t is measured in minutes after 8 am.

(i) Find the velocity of L relative to B during these 25 minutes. *(2 marks)*

(ii) Find an expression for the distance, in metres, which the lion and the hot air balloon are apart at time t, where $0 < t < 25$. You do **not** need to simplify your expression. *(2 marks)*

(iii) Hence find the time at which the lion and the balloon are nearest to each other. *(4 marks)*

Final Challenge

The unit vectors **i** and **j** are defined in the east and north directions respectively. The unit of distance is kilometres and the unit of velocity is kilometres per hour.

Initially, two ships P and Q are 2 kilometres apart with P due south of Q.

Ship Q is travelling with velocity $10\sqrt{3}\mathbf{i} - 10\mathbf{j}$ kilometres per hour.

The maximum speed of ship P is 8 kilometres per hour.

(a) Find the speed of ship Q, and the bearing on which it is travelling. *(3 marks)*

(b) Ship P travels to ensure that it approaches Q as closely as possible.

(i) Find the direction in which P travels. *(4 marks)*

(ii) Show that the velocity of Q relative to P is $11\mathbf{i} - 15\mathbf{j}$ correct to 2 significant figures. *(3 marks)*

(iii) Find the shortest distance between the ships. *(4 marks)*