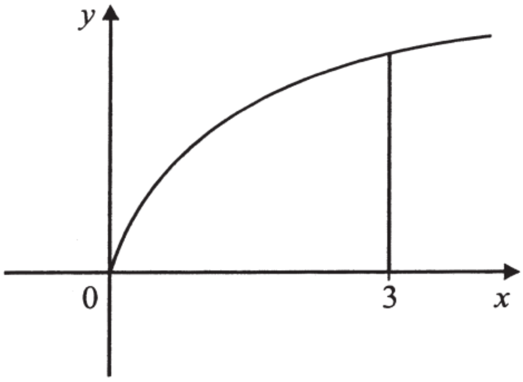

The diagram shows a sketch of the curve $y = 2\sqrt{x}$.

The arc of the curve between $x = 0$ and $x = 3$ is rotated through 2π radians about the x – axis.

(a) Show that S, the surface area generated, is given by

$$S = 4\pi \int_0^3 \sqrt{1 + x}\, dx\,.$$ *(5 marks)*

(b) Hence evaluate S. *(3 marks)*

Q	Solution	Marks	Total	Comments
3 (a)	$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$	B1		
	$\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)} = \sqrt{\left(1+\frac{1}{x}\right)} = \left(\sqrt{\left(\frac{x+1}{x}\right)}\right)$	M1A1√		ft incorrect B1
	$S = \int_0^3 2\pi\, 2\sqrt{x}\sqrt{\frac{x+1}{x}}\, dx$	M1		
	$= 4\pi \int_0^3 \sqrt{x+1}\, dx$	A1	5	AG; CAO
(b)	$S = (4\pi)\frac{2}{3}\left[(x+1)^{\frac{3}{2}}\right]_0^3$	M1A1		For the M1 the integral must be of the form $k(1+x)^{\frac{3}{2}}$. The A1 is for the correct coefficient
	$= \frac{56\pi}{3}$ (58.64)	A1√	3	Accept answer to 1 DP
	Total		**8**	

A curve has equation

$$y = \sinh^2 x.$$

(a) Show that

$$1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2 2x. \qquad \textit{(2 marks)}$$

The arc of the curve between $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis.

(b) (i) Show that S, the area of the curved surface generated, is given by

$$S = \pi \int_0^1 (\cosh 2x - 1) \cosh 2x \, dx. \qquad \textit{(3 marks)}$$

(ii) Hence find S, giving an exact answer in terms of hyperbolic functions. *(4 marks)*

5 (a)	$\frac{dy}{dx} = 2\sinh x \cosh x$	B1		
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	B1	2	
(b)(i)	$S = 2\pi \int_0^1 \sinh^2 x \cosh 2x \, dx$	M1		Do not insist on limits at this stage
	Use of $\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$	m1		Clearly used
	$S = \pi \int_0^1 (\cosh 2x - 1) \cosh 2x \, dx$	A1	3	AG
(ii)	Attempt to put $\cosh^2 2x$ in terms of $\cosh 4x$	M1		If attempted by integration by parts, must be able to handle $\int \sinh^2 2x$ for M1. Then A2,1,0 for integration
	$S = \pi \int_0^1 \left[\frac{1}{2}(1 + \cosh 4x) - \cosh 2x\right] dx$	A1		
	$= \pi\left[\frac{x}{2} + \frac{\sinh 4x}{8} - \frac{\sinh 2x}{2}\right]$	A1F		
	$= \pi\left[\frac{1}{2} + \frac{\sinh 4}{8} - \frac{\sinh 2}{2}\right]$	A1F	4	
	Total		**9**	

A curve C has equation

$$y = \ln(1 - x^2), \qquad 0 \leqslant x < 1.$$

(a) Show that

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{1 + x^2}{1 - x^2}\right)^2.$$ *(6 marks)*

(b) Use the result

$$\frac{1 + x^2}{1 - x^2} = \frac{2}{1 - x^2} - 1$$

to show that the length of the arc of C between the points where $x = 0$ and $x = p$ is

$$2\tanh^{-1} p - p.$$ *(4 marks)*

Q	Solution	Marks	Total	Comments
5 (a)	$\frac{dy}{dx} = \frac{-2x}{1-x^2}$	B1, B1		B1 each numerator and denominator
	$1+\left(\frac{dy}{dx}\right)^2 = 1 + \frac{4x^2}{(1-x^2)^2}$	M1		
	$= \frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2}$	A1F		
	$= \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}$	A1		CAO
	$= \left(\frac{1+x^2}{1-x^2}\right)^2$	A1	6	
(b)	arc length $= \int_0^p \left(\frac{1+x^2}{1-x^2}\right) dx$	M1		
	$= \int_0^p \left(\frac{2}{1-x^2} - 1\right) dx$	A1		
	$\left[2\tanh^{-1} x - x\right]_0^p$	A1F		ft if hyperbolic
	$= 2\tanh^{-1} p - p$	A1	4	AG
	Total		**10**	

(a) Evaluate:

(i) $\int \cosh^2 x \, dx$; *(3 marks)*

(ii) $\int x \cosh x \, dx$. *(3 marks)*

(b) A curve C is given parametrically by the equations

$$x = \cosh t + t, \qquad y = \cosh t - t.$$

Express

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

in terms of $\cosh t$. *(5 marks)*

(c) (i) The arc of C from $t = 0$ to $t = 1$ is rotated through 2π radians about the x-axis.

Show that S, the area of the curved surface generated, is given by

$$S = 2\pi\sqrt{2}\int_0^1 (\cosh t - t)\cosh t \, dt.$$ *(1 mark)*

(ii) Hence find S, leaving your answer in terms of hyperbolic functions. *(4 marks)*

Q	Solution	Marks	Total	Comments
4 (a)(i)	$\int \cosh^2 x\,dx = \int \frac{1}{2}(1+\cosh 2x)dx$	M1A1		
	$= \frac{x}{2} + \frac{\sinh 2x}{4} (+c)$	A1F	3	
(ii)	$\int x\cosh x\,dx = x\sinh x - \int \sinh x\,dx$	M1A1		if $\int \sinh x$ is given as $-\cosh x$, penalise once only if consistent
	$= x\sinh x - \cosh x(+c)$	A1F	3	
(b)	$\dot{x} = \sinh t + 1, \quad \dot{y} = \sinh t - 1$	B1		
	$\dot{x}^2 + \dot{y}^2 = (\sinh t+1)^2 + (\sinh t-1)^2$	M1		
	$= \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1$	A1F		
	Use of $\cosh^2 t - \sinh^2 t = 1$	m1		
	$= 2\cosh^2 t$	A1F	5	
(c)(i)	$S = \int_0^1 2\pi y\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{1}{2}} dt$			
	$= 2\pi\sqrt{2}\int_0^1 (\cosh t - t)(\cosh t)\,dt$	B1	1	
(ii)	$= 2\sqrt{2}\pi\left[\frac{t}{2} + \frac{\sinh 2t}{4} - t\sinh t + \cosh t\right]_0^1$	M1A1		
	$= 2\sqrt{2}\pi\left\{\left[\frac{1}{2} + \frac{1}{4}\sinh 2 - \sinh 1 + \cosh 1\right] - (+1)\right\}$			
	$= 2\sqrt{2}\pi\left[\frac{1}{4}\sinh 2 - \sinh 1 + \cosh 1 - \frac{1}{2}\right]$	A2,1,0F	4	must use $\cosh 0 = 1$ and $\sinh 0 = 0$ for A2, otherwise A1
	Total		**16**	