FP2 - Roots of Polynomials

Challenge 1

One of the roots of the cubic equation

$$x^3 + kx - 80 = 0,$$

where k is real, is $-2 + 4\text{i}$. Find the other two roots and the value of k. *(5 marks)*

Challenge 2

The cubic equation

$$x^3 + 2px^2 - 8 = 0, \qquad \text{where } p \text{ is real,}$$

has roots α, β and $\alpha + \beta$.

(a) Show that:

(i) $\alpha + \beta = -p$; *(2 marks)*

(ii) $\alpha\beta = -\dfrac{8}{p}$. *(2 marks)*

(b) Show that $p = 2$. *(5 marks)*

Challenge 3

The roots of the cubic equation

$$x^3 - 6x^2 + 4px - p^2 = 0$$

are $\beta - k$, β and $\beta + k$.

(a) Show that

(i) $\beta = 2$, *(2 marks)*

(ii) $k^2 = 4\,(3 - p)$, *(3 marks)*

(iii) $k^2 = 4 - \frac{1}{2}p^2$. *(2 marks)*

(b) Hence find

(i) the value of p, *(3 marks)*

(ii) the two non-real roots of the cubic equation giving your answers in the form $a + \mathrm{i}b$, where a and b are real. *(3 marks)*

Final Challenge

(a) (i) Express $(1 - i)(3 - i)$ in the form $a + ib$, where a and b are real. *(2 marks)*

(ii) Show that

$$(1 - i)^3 = -2 - 2i.$$

(2 marks)

(b) (i) Show that $1 - i$ is a root of the cubic equation

$$z^3 + iz^2 - (3 - i)z + 2(1 - i) = 0.$$

(3 marks)

(ii) Given that the other two roots of the cubic equation are α and β, show that

$$\alpha + \beta = -1$$

and find $\alpha\beta$. *(3 marks)*

(c) Hence solve the cubic equation completely. *(3 marks)*