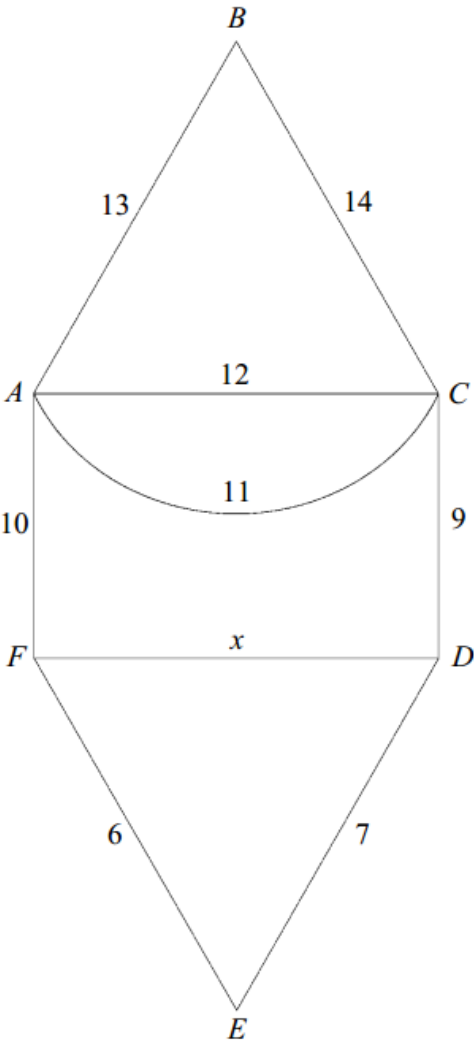


The following diagram shows a network of roads connecting six towns. The number on each arc represents the distance, in miles, between towns. The road connecting towns D and F has length x miles, where $x < 13$.

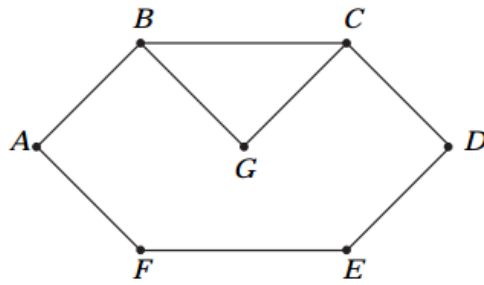


An optimal Chinese Postman route, starting and finishing at A , has length 100 miles. Find the value of x . (4 marks)

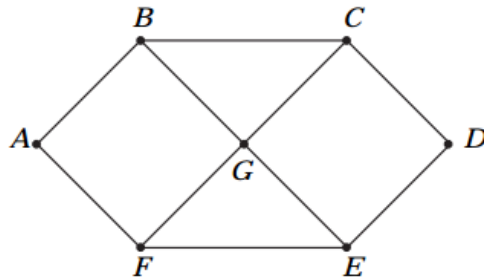
2	Odd vertices D and F Repeat x or 13 (or DF) $\therefore 2x + 82 = 100$ $x = 9$	E1 B1 M1 A1	4	} May be implied
		Total	4	

3 The following question refers to the three graphs: **Graph 1**, **Graph 2** and **Graph 3**.

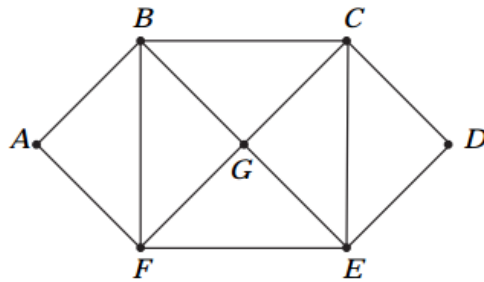
Graph 1



Graph 2



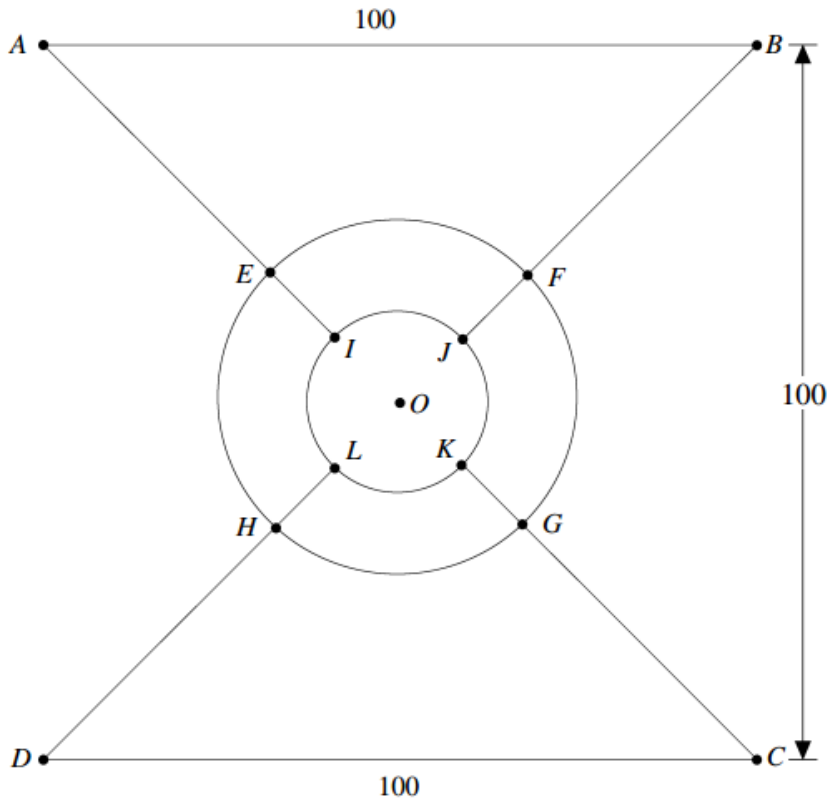
Graph 3



- (a) For **each** of the graphs explain whether or not the graph is Eulerian. (4 marks)
- (b) The length of each edge connecting two vertices is 1 unit. Find, for **each** of the graphs, the length of an optimal Chinese postman route, starting and finishing at A. (4 marks)

Q	Solution	Marks	Total	Comments	
3	(a) Graph 1: no – odd vertices (semi) Graph 2: no – odd Graph 3: yes – even	M1B1 B1 B1	4	for considering vertices	
	(b) Graph 1: 8 edges + <u>1</u> repeat (BC) <u>9</u>	M1 B1			for considering repeats on 1 or 2 allow listings of CP.
	Graph 2: 10 edges + <u>2</u> repeats (BC and FE) <u>12</u>	B1			
Graph 3: 12 edges + <u>0</u> repeats <u>12</u>	B1	4			
Total			8		

The following diagram shows the paths in St. Stephen's Green, Dublin. The paths AB and CD are parallel and are each 100 metres long. The perpendicular distance between them is also 100 metres. The point O is equidistant from A , B , C and D . The path $IJKL$ is a circle, centre O , of radius 20 metres, and the path $EFGH$ is a circle, centre O , of radius 40 metres.



A person enters the park at A and is to walk along each path at least once before returning to A .

(a) Giving each answer to the nearest metre, calculate the lengths of the paths:

(i) IJ ; (1 mark)

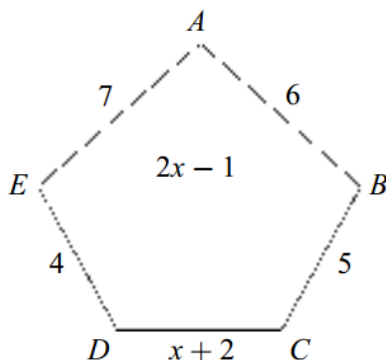
(ii) EF ; (1 mark)

(iii) AE . (2 marks)

(b) Calculate the length of an optimal Chinese Postman tour. (5 marks)

Q	Solution	Marks	Total	Comments
5 (a)(i)	$IJ = 31$	B1	1	In part (a), if answers given to more accuracy than nearest metre, deduct 1 only.
	(ii) $EF = 63$	B1	1	
(iii)	$AC = \sqrt{(100^2 + 100^2)} (= 141.)$ $AE = 31$	M1 A1	2	
(b)	Odd vertices I, J, K, L	B1		ft 2 × their (a) (i) (their 62) M1 for 4×, 4×, 4×, 2×, 4× or 843 AWRT
	Min $31 + 31 = 62$	B1F		
	Total = each path + 62	M1		
	$= 31 \times 4 = 124$	M1		
	$63 \times 4 = 252$			
	$31 \times 4 = 124$			
	$100 \times 2 = 200$			
$20 \times 4 = 80$				
	<hr/>			
	780			
	+ 62			
	<hr/>			
	842	A1	5	
	AWRT			

The following diagram shows a network of roads connecting five villages. The numbers on the roads are the times, in minutes, taken to travel along each road, where $x > 0.5$.



A police patrol car has to travel from its base at B along each road at least once and return to base.

(a) Explain why a route from B to E must be repeated. (1 mark)

(b) List the routes, and their lengths, from B to E , in terms of x where appropriate. (2 marks)

(c) On a particular day, it is known that $x = 10$.

Find the length of an optimal Chinese Postman route on this day. State a possible route corresponding to this minimum length. (4 marks)

(d) Find, no matter what the value of x , which of the three routes should **not** be used if the total length of a Chinese Postman route is to be optimal. (5 marks)

Q	Solution	Marks	Total	Comments
6 (a)	Odd vertices	E1	1	
(b)	$BE = 2x - 1$ $BAE = 13$ $BCDE = x + 11$	B2, 1, 0	2	
(c)	$x = 10 \therefore$ repeat BAE Total = $53 + 13 = 66$ Route eg $BAEDCBEAB$	B1 M1A1 B1	4	for 53+ OE (Must have 9 letters)
(d)	$BE = 2x - 1 < 13 \quad x < 7$ $2x - 1 < x + 11 \quad x < 12$ $BAE = 13 < 2x - 1 \quad x > 7$ $13 < x + 11 \quad x > 2$ $BCDE = x + 11 < 2x - 1 \quad x > 12$ $x + 11 < 13 \quad x < 2$ \therefore not $BCDE$	M1 A1 M1 A1 E1	5	SC: Substitute values Either 1/5 (no explanation of values) or M2A2E1 complete table with $x = 1$ to $x = 11$ or M1A1 $x \leq 2 \Rightarrow BE < BCDE$ M1A1 $x > 2 \Rightarrow BAE < BCDE$ E1 \therefore Not $BCDE$
	Total		12	
	Total		60	