Sequences and Series

The first four terms of a geometric sequence are

$$10,\ 9,\ 8.1,\ 7.29\,.$$

(a) Show that the common ratio of the sequence is 0.9. *(1 mark)*

(b) Find the nth term. *(2 marks)*

(c) Show that the sum of the first 25 terms is approximately 92.8. *(2 marks)*

(d) Find the sum to infinity. *(2 marks)*

Q	Solution	Marks	Total	Comments
1 (a)	$10r = 9 \Rightarrow r = 0.9$	B1	1	Convincingly shown (AG)
(b)	Formula for nth term of GP stated	M1		Or used
	$u_n = 10(0.9)^{n-1}$	A1	2	OE
(c)	Formula for sum to n terms stated	M1		Or used; M0 for list of terms
	$S_{25} = \dfrac{10(1-0.9^{25})}{1-0.9} \approx 92.8(21)$	A1	2	AG (92.8): allow just 3SF if no error
(d)	Formula for sum to infinity stated	M1		Or used
	$S_\infty = 100$	A1	2	
	Total		**7**	

The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

Question Number	Scheme	Marks
9 (a)	Initial step: Two of: $a = k+4,\ ar = k,\ ar^2 = 2k-15$ Or one of: $r = \frac{k}{k+4},\quad r = \frac{2k-15}{k},\quad r^2 = \frac{2k-15}{k+4}$, Or $k = \sqrt{(k+4)(2k-15)}$ or even $k^3 = (k+4)k(2k-15)$ $k^2 = (k+4)(2k-15)$, so $k^2 = 2k^2 + 8k - 15k - 60$ Proceed to $k^2 - 7k - 60 = 0$ (*)	M1 M1, A1 A1 (4)
(b)	$(k-12)(k+5) = 0 \qquad k = 12$ (*)	M1 A1 (2)
(c)	Common ratio: $\frac{k}{k+4}$ or $\frac{2k\text{-}15}{k} = \frac{12}{16}\left(= \frac{3}{4} \text{ or } 0.75\right)$	M1 A1 (2)
(d)	$\frac{a}{1-r} = \frac{16}{\left(\frac{1}{4}\right)} = 64$	M1 A1 (2) [10]

The first term of an arithmetic series is 7. The tenth term is 43.

(a) Find the common difference. *(2 marks)*

(b) Find the sum of the first fifty terms of the series. *(3 marks)*

(c) The kth term has a value greater than 1000.

(i) Show that $4k > 997$. *(2 marks)*

(ii) Find the least possible value of k. *(1 mark)*

Question Number and part	Solution	Marks	Total	Comments
1(a)	$7+9d$	M1		Condone $7+10d$, or attempt to consider $\frac{43-7}{9}$ {or 10}
	$7+9d = 43 \qquad d = 4$	A1	2	
(b)	$S_n = \frac{1}{2}n(2a+(n-1)d)$ formula attempted	M1		Condone one slip using n or 50
	$= 25\ (14+196)$	A1√		ft their 49 d (eg $25 \times 190.4 = 4760$)
	$= 5250$	A1	3	
(c)(i)	$7+(k-1)d > 1000$	M1		Condone = instead of >
	$\Rightarrow 4k > 997$	A1	2	**ag** be convinced
(ii)	(Since k is integer) $\qquad k = 250$	B1	1	
	Total		**8**	

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \quad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

Question Number		Scheme	Marks	
Q7	(a)	$(a_2 =)2k-7$	B1	(1)
	(b)	$(a_3 =)2(2k-7)-7$ or $4k-14-7, = 4k-21$ (*)	M1, A1cso	
				(2)
	(c)	$(a_4 =)2(4k-21)-7 \quad (=8k-49)$	M1	
		$\sum_{r=1}^{4} a_r = k + "(2k-7)" + (4k-21) + "(8k-49)"$	M1	
		$k+(2k-7)+(4k-21)+(8k-49) = 15k-77 = 43 \qquad k = 8$	M1 A1	(4)
				[7]