Core 2 – Sequences and Series

Challenge 1

The first four terms of a geometric sequence are

$$10, \ 9, \ 8.1, \ 7.29.$$

(a)   Show that the common ratio of the sequence is 0.9.   *(1 mark)*

(b)   Find the nth term.   *(2 marks)*

(c)   Show that the sum of the first 25 terms is approximately 92.8.   *(2 marks)*
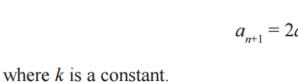
(d)   Find the sum to infinity.   *(2 marks)*

Challenge 2

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\displaystyle\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

Challenge 3

The first term of an arithmetic series is 7.   The tenth term is 43.

(a)   Find the common difference.   *(2 marks)*

(b)   Find the sum of the first fifty terms of the series.   *(3 marks)*

(c)   The kth term has a value greater than 1000.

   (i)   Show that $4k > 997$.   *(2 marks)*

   (ii)   Find the least possible value of k.   *(1 mark)*

Final Challenge

The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)