1 (a) Find the sum of the integers from 1 to 300 inclusive. *(3 marks)*

(b) Evaluate

$$\sum_{r=1}^{300} (r^2 + r) .$$ *(3 marks)*

Question	Solution	Marks	Total	Comments
1 (a)	AP $a = 1,\ d = 1, n = 300, l = 300$	M1		Recognition of AP down to a formula for the sum involving a maximum of one unknown
	$S = \frac{300}{2}(1+300)$ OE	A1		
	$= 45150$	A1	**(3)**	
(b)	$\ldots. = \sum_{1}^{300} r^2 + \sum_{1}^{300} r$	M1		Valid 1st step splitting the terms in r
	$= \frac{300}{6}(300+1)(2\times300+1) + [\text{ans.(a)}]$	A1 ft		ft on cand's answer to (a) if not correct
	$\ldots = 9\ 045\ 050 + 45150$	A1	**(3)**	
		TOTAL	**(6)**	

3 (a) Use the formulae

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

and

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} r^2(r-1) = \tfrac{1}{12}n(n^2-1)(3n+2)$$ *(4 marks)*

(b) Use the result from part (a) to find the value of

$$\sum_{r=4}^{11} r^2(r-1)$$ *(3 marks)*

3(a)	$\Sigma r^2(r-1) = \Sigma r^3 - \Sigma r^2$	M1		With attempt to use the given formulae
	Good progress with expansion	m1		or use of common factors
	Factors n and $n+1$ found	A1		Allow verification here
	$\ldots = \dfrac{1}{12}n(n^2-1)(3n+2)$	A1	4	Convincingly shown (AG)
(b)	Use of f(11) – f(3) in above	M1		M1 for f(11) – f(4)
	f(11) = 3850	A1		PI by correct answer
	f(3) = 22 (so answer is 3828)	A1	3	ditto
	Total		**7**	

3 (a) Find the value of:

(i) $\sum_{r=1}^{100} r^3$ *(1 mark)*

(ii) $\sum_{r=51}^{100} r^3$ *(2 marks)*

(b) Find the sum of the fifty integers from 51 to 100 inclusive. *(3 marks)*

(c) Hence find the value of $\sum_{r=51}^{100} (r^3 - 6325r)$. *(2 marks)*

Question Number and part	Solution	Marks	Total	Comments
3(a)(i)	25 502 500	B1	1	
(ii)	Attempt to find $S_{100} - S_{50}$ using $\sum r^3$ = 23 876 875	M1 A1	2	Formula in booklet. Condone $S_{100} - \begin{cases} S_{51} \\ S_{49} \end{cases}$
(b)	$S_n = \frac{1}{2}n(2a + (n-1)d)$ formula attempted (condone one slip)	M1		**Or** $\frac{n(\text{first} + \text{last})}{2}$ attempted **Or** $S_{100} - S_{50/\ 51/\ 49}$ using $\sum r = \frac{1}{2}n(n+1)$
	correct values substituted, candidate's 25 (51+100)	m1		**Or** candidate's $50 \times 101 - 25 \times 51$
	= 3775	A1	3	**sc** B3 for correct answer without working **sc** B2 for correct answer without working
(c)	Use of (a)(ii) – 6325 (b) = 0	M1 A1	2	
	Total		**8**	

5 (a) Use the identity

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} (r^3 - 1) = \tfrac{1}{4}n(n-1)(n^2+3n+4)$$ *(4 marks)*

(b) Hence show that $\sum_{r=1}^{9} (r^3 - 1)$ is divisible by 7 . *(2 marks)*

5(a)	$(n+1)^2 = n^2 + 2n + 1$	B1		
	$\Sigma(r^3 - 1) = (\Sigma r^3) - n$	M1		
	$\ldots = \tfrac{1}{4}n(n^3 + 2n^2 + n - 4)$	A1		oe
	Hence result	A1	4	ag convincingly shown
(b)	$n = 9 \Rightarrow \tfrac{1}{4}(n^2 + 3n + 4) = 28$	M1		
	$\ldots \Rightarrow$ expression = $7(4\times9\times8)$, Hence result	A1	2	ag convincingly shown
	Total		**6**	