# FP1 - Series Challenge

## Challenge 1

(a) Find the sum of the integers from 1 to 300 inclusive. *(3 marks)*

(b) Evaluate

$$\sum_{r=1}^{300} (r^2 + r) .$$

*(3 marks)*

# Challenge 2

(a) Use the formulae

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

and $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} r^2(r-1) = \frac{1}{12}n(n^2-1)(3n+2)$$ *(4 marks)*

(b) Use the result from part (a) to find the value of

$$\sum_{r=4}^{11} r^2(r-1)$$ *(3 marks)*

# Challenge 3

(a) Find the value of:

(i) $\sum_{r=1}^{100} r^3$ *(1 mark)*

(ii) $\sum_{r=51}^{100} r^3$ *(2 marks)*

(b) Find the sum of the fifty integers from 51 to 100 inclusive. *(3 marks)*

(c) Hence find the value of $\sum_{r=51}^{100} (r^3 - 6325r)$. *(2 marks)*

# Final Challenge

(a) Use the identity

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

to show that

$$\sum_{r=1}^{n} (r^3 - 1) = \tfrac{1}{4}n(n-1)(n^2 + 3n + 4)$$ *(4 marks)*

(b) Hence show that $\sum_{r=1}^{9} (r^3 - 1)$ is divisible by 7 . *(2 marks)*