(a) Show that

$$\frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}.$$

(2 marks)

(b) Hence find

$$\sum_{r=1}^{n} \frac{r}{(r+1)!}.$$

(3 marks)

Q		Solution	Marks	Total	Comments
5	**(a)**	Use of $(r+1)! = r! \times (r+1)$	M1		
		Result	A1	2	
	(b)	$r=1 \quad \frac{1}{2!} = \frac{1}{1!} - \frac{1}{2!}$			
		$r=2 \quad \frac{2}{3!} = \frac{1}{2!} - \frac{1}{3!}$			
		$r=n \quad \frac{n}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!}$			
		three lines set out (including the last row)	B1		
		Cancellations clearly shown	M1		
		Sum $= 1 - \frac{1}{(n+1)!}$	A1	3	
		Total		**5**	

Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}.$$

(8 marks)

6	Assume true for $n = k$ Then $\sum_{r=1}^{k+1} \frac{1}{(3r-2)(3r+1)} = \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$	M1A1		
	$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$	A1√		
	$= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$	M1A1√		Factorisation of numerator
	$= \frac{k+1}{3(k+1)+1}$ or $\frac{k+1}{3k+4}$	A1		
	$\therefore$ if result is true for $n = k$, it is true for $n = k+1$			
	but $n = 1, LHS = \frac{1}{1 \times 4} = \frac{1}{4} = RHS$	B1		
	$\therefore$ true for $n \geq 1$ by induction	E1	8	E1 is for an acceptable formal outline
	In Q6 using the difference method Partial fractions Writing out series in detail (including 1st 2 and last terms) Answer **Total** $4/8$	*(M1A1)* *(A1)* *(A1)*		 CAO
	Total		**8**	

(a) Show that $\frac{2r-1}{(r-1)r} - \frac{2r+1}{r(r+1)} \equiv \frac{2}{(r-1)(r+1)}$. *(3 marks)*

(b) Hence, using the method of differences, prove that

$$\sum_{r=2}^{n} \frac{2}{(r-1)(r+1)} = \frac{3}{2} - \frac{2n+1}{n(n+1)}$$ *(3 marks)*

(c) Deduce the sum of the infinite series

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \ldots + \frac{1}{(n-1)(n+1)} + \ldots$$ *(2 marks)*

9(a)	$\dfrac{(2r-1)(r+1)-(2r+1)(r-1)}{r(r-1)(r+1)}$	M1		Common denominator attempt
	$=\dfrac{2r}{r(r-1)(r+1)}$	m1		Multiplying out brackets and cancelling
	$=\dfrac{2}{(r-1)(r+1)}$	A1	3	**ag**
(b)	$=\left(\dfrac{3}{1\times2}-\dfrac{5}{2\times3}\right)+\left(\dfrac{5}{2\times3}-\dfrac{7}{3\times4}\right)+\ldots$	M1		Putting $r=2,3,\ldots$
	$\dfrac{2n-1}{(n-1)n}-\dfrac{2n+1}{n(n+1)}$	m1		nth term and terms cancelling
	sum $=\dfrac{3}{2}-\dfrac{(2n+1)}{n(n+1)}$	A1	3	**ag** Be convinced – all correct
(c)	$\dfrac{(2n+1)}{n(n+1)}\to 0$ as $n\to\infty$	M1		
	Sum of infinite series $=\dfrac{3}{4}$	A1	2	
	Total		**8**	

The function f is given by

$$f(n)=n^3+(n+1)^3+(n+2)^3.$$

(a) Simplify, as far as possible, $f(n+1)-f(n)$. *(4 marks)*

(b) Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. *(5 marks)*

Q	Solution	Marks	Total	Comments
3 (a)	$f(n+1)-f(n)=(n+3)^3-n^3$	M1A1		or attempt at $f(n+1)-f(n)$ M1
	$=n^3+3n^2\times3+3n\times9+27-n^3$	A1		$3n^3+18n^2+42n+36$ A1
	$=9n^2+27n+27$	A1F	4	$3n^3+9n^2+15n+9$ A1 result A1
(b)	Assume result true for $n=k$ ie $f(k)=M(9)$ Then $f(k+1)=f(k)+M(9)$ $=M(9)+M(9)=M(9)$	M1A1		
	But $f(1)=1^3+2^3+3^3=36=M(9)$	A1		Must be clear for this A1
	P_1 true and $P_k\Rightarrow P_{k+1}$ $\therefore$ true by induction	B1 E1	5	Only if correct or almost correct
	Total		**9**	