FP2 - Summations and proof

Challenge 1

(a) Show that

$$\frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}.$$

(2 marks)

(b) Hence find

$$\sum_{r=1}^{n} \frac{r}{(r+1)!}.$$

(3 marks)

Challenge 2

Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}.$$

(8 marks)

Challenge 3

(a) Show that $\frac{2r-1}{(r-1)r} - \frac{2r+1}{r(r+1)} \equiv \frac{2}{(r-1)(r+1)}$. *(3 marks)*

(b) Hence, using the method of differences, prove that

$$\sum_{r=2}^{n} \frac{2}{(r-1)(r+1)} = \frac{3}{2} - \frac{2n+1}{n(n+1)}$$

(3 marks)

(c) Deduce the sum of the infinite series

$$\frac{1}{1\times 3} + \frac{1}{2\times 4} + \frac{1}{3\times 5} + \ldots + \frac{1}{(n-1)(n+1)} + \ldots$$

(2 marks)

Final Challenge

The function f is given by

$$f(n) = n^3 + (n+1)^3 + (n+2)^3.$$

(a) Simplify, as far as possible, $f(n+1) - f(n)$. *(4 marks)*

(b) Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. *(5 marks)*