

Surds

Express $(4 - \sqrt{7})(5 + 2\sqrt{7})$ in the form $a + b\sqrt{7}$, where a and b are integers. (3 marks)

(a) Express each of the following in the form $k\sqrt{5}$:

(i) $\sqrt{45}$

(ii) $\frac{20}{\sqrt{5}}$

(3 marks)

(b) Hence write $\sqrt{45} + \frac{20}{\sqrt{5}}$ in the form $n\sqrt{5}$, where n is an integer. (1 mark)

1	$20 - 5\sqrt{7} + 8\sqrt{7} - 2\sqrt{7}\sqrt{7}$ ← 14 = $6 + 3\sqrt{7}$	M1 B1 A1	3	At least 3 terms
	Total		3	

Question Number and part	Solution	Marks	Total	Comments
1(a)(i)	$3\sqrt{5}$	B1		
(ii)	$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ Ans = $4\sqrt{5}$	M1 A1	3	Clear attempt to rationalise denominator Eg. $\times \frac{\sqrt{5}}{\sqrt{5}}$ sc B2 for correct answer sc B1 for 8.9442... from calculator $\Rightarrow k = 4$
(b)	$7\sqrt{5}$	B1✓	1	fit their answers from (a) - must be integer
	Total		4	

(a) Express $(\sqrt{7} + 1)^2$ in the form $a + b\sqrt{7}$, where a and b are integers. (2 marks)

(b) Hence express $\frac{(\sqrt{7} + 1)^2}{(\sqrt{7} + 2)}$ in the form $p + q\sqrt{7}$, where p and q are rational numbers. (3 marks)

Question	Solution	Marks	Total	Comments
3 (a)	$\dots = (\sqrt{7})^2 + 2\sqrt{7} + 1$ $\dots = 8 + 2\sqrt{7}$	M1 A1	(2)	Expansion to 3 terms or better Accept 'a = 8, b = 2'
(b)	$\dots = \frac{(8 + 2\sqrt{7})(\sqrt{7} - 2)}{(\sqrt{7} + 2)(\sqrt{7} - 2)}$ $\dots = \frac{4\sqrt{7} - 2}{3}$	M1 A1 A1		(3)
		TOTAL	(5)	

Express each of the following in the form $p + q\sqrt{3}$:

(a) $(2 + \sqrt{3})(5 - 2\sqrt{3})$; (3 marks)

(b) $\frac{26}{4 - \sqrt{3}}$. (3 marks)

3(a)	$10 - 4\sqrt{3} + 5\sqrt{3} - 2(\sqrt{3})^2$ <p style="text-align: right; margin-right: 20px;"> \nwarrow 6 or 2×3 $= 4 + \sqrt{3}$ </p>	M1 B1 A1	3	At least 3 terms not necessarily correct Implied by $4 + k\sqrt{3}$ or $16 + k\sqrt{3}$
(b)	$\frac{26}{4 - \sqrt{3}} \times \frac{4 + \sqrt{3}}{4 + \sqrt{3}}$ <p style="text-align: right; margin-right: 20px;"> \searrow denominator = 13 $= 8 + 2\sqrt{3}$ </p>	M1 A1 A1		
Total			6	

(a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant. (1)

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm².

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found. (4)

Question Number	Scheme	Marks
6.	<p>(a) $80 = 5 \times 16$ $\sqrt{80} = 4\sqrt{5}$</p> <p>Method 1</p> <p>(b) $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$</p> <p>$= \frac{\sqrt{80}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ or $\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$</p> <p>$= \frac{20-4\sqrt{5}}{4}$ or $\frac{4\sqrt{5}-20}{-4}$</p> <p>$= 5-\sqrt{5}$</p> <p>Method 2</p> <p>$(p+q\sqrt{5})(\sqrt{5}+1) = \sqrt{80}$</p> <p>$p\sqrt{5}+q\sqrt{5}+p+5q = 4\sqrt{5}$</p> <p>$p+5q=0$ $p+q=4$ $p=5, q=-1$</p>	<p>B1 (1)</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1cao</p> <p>(4) (5 marks)</p>