FP4 Systems of Equations Challenge

Challenge 1

The equations

$$x + y - 2z = 2$$
$$3x - y + 6z = 2$$
$$6x + 5y - 9z = k$$

represent three planes, where k is a constant.

(a) Show that this system of equations does not have a unique solution. *(2 marks)*

(b) Prove that this system is consistent provided $k = 11$. *(4 marks)*

(c) (i) Find the solution to this system in the case when $k = 11$. *(4 marks)*

(ii) Interpret this solution with reference to the three planes. *(1 mark)*

Challenge 2

Three simultaneous equations are

$$\begin{aligned} x - 3y + 2z &= 3 \\ x + y + az &= b \\ x - 2y + z &= 2, \end{aligned}$$

where a and b are constants.

(a) In the case where $a \neq -2$, solve the equations in terms of a and b. *(7 marks)*

(b) Give, with reasons, a geometrical interpretation of the planes represented by these three equations in the case where $a = -2$ and $b \neq -1$. *(3 marks)*

Challenge 3

A matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} 3 & 1 & 8 \\ 2 & -1 & 5 \\ 1 & 2 & a \end{bmatrix}.$$

(a) Find det $\mathbf{M}$ in terms of a. *(3 marks)*

(b) Find the value of a for which the matrix $\mathbf{M}$ is singular. *(1 mark)*

(c) (i) In the case $a = 2$, find $\mathbf{M}^{-1}$. *(6 marks)*

(ii) Hence, or otherwise, solve

$$\begin{aligned} 3x + y + 8z &= 3 \\ 2x - y + 5z &= 0 \\ x + 2y + 2z &= 2. \end{aligned}$$

(4 marks)

Final Challenge

Two planes are represented by the equations

$$\begin{aligned} x + y + z &= 3, \\ 5x + y + 3z &= 29. \end{aligned}$$

(a) Find the equations of the line of intersection of the planes, giving your answer in the form

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}.$$

(6 marks)

(b) Show that this line also lies in the plane with equation

$$4x - 2y + z = 33.$$

(3 marks)