Trapezium rule

Use the trapezium rule with five ordinates (four strips) to find an approximation to

$$\int_1^3 \frac{1}{x^3+3}\,dx$$

giving your answer to 3 significant figures. *(4 marks)*

Question Number and part	Solution	Marks	Total	Comments
1	$h = 0.5$	B1		
	Integral $= \frac{h}{2}\{\ldots\}$			
	$\{\ldots\} = \left[\frac{1}{4} + \frac{1}{30} + 2\left(\frac{8}{51} + \frac{1}{11} + \frac{8}{149}\right)\right]$	M1 A1		At least 3 terms correct 5 terms, at least 4 correct
	Integral = 0.222	A1	4	cao must be 0.222
	sc (for 5 strips) $h = 0.4$			B0 M1 at least 4 terms correct A1 6 terms at least 5 correct A1cao
	Total		**4**	

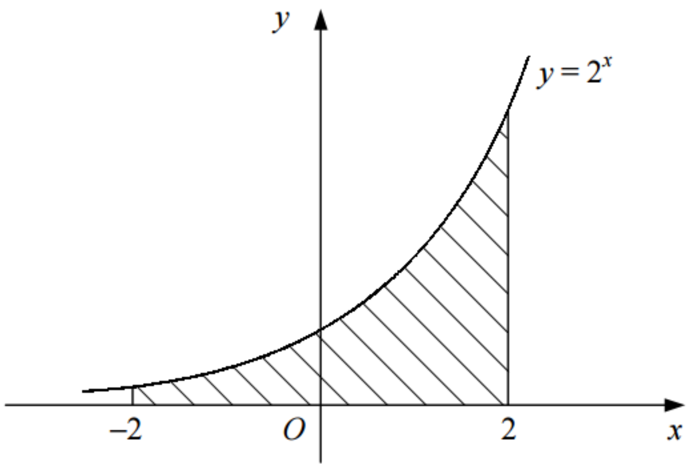

Figure 1

Figure 1 shows the curve with equation $y = 2^x$.

Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region bounded by the curve, the x-axis and the lines $x = -2$ and $x = 2$. **(5)**

x	-2	-1	0	1	2	
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	B1

area $\approx \frac{1}{2} \times 1 \times [\frac{1}{4} + 4 + 2(\frac{1}{2} + 1 + 2)]$ B1 M1 A1

$= 5\frac{5}{8}$ or 5.63 (3sf) A1 **(5)**

$$y = \sqrt{(10x - x^2)}.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\,dx$.

(4)

3 (a)	3.84, 4.14, 4.58 (Any one correct B1 B0. All correct B1 B1)	B1 B1 (2)
(b)	$\frac{1}{2}\times 0.4,\ \{(3+4.58)+2(3.47+3.84+4.14+4.39)\}$ = 7.852 (awrt 7.9)	B1, M1 A1ft A1 (4) **[6]**
Notes (a)	**B1** for one answer correct Second **B1** for all three correct Accept awrt ones given or exact answers so $\sqrt{21}$, $\sqrt{\left(\frac{369}{25}\right)}$ or $\frac{3\sqrt{41}}{5}$, and $\sqrt{\left(\frac{429}{25}\right)}$ or $\frac{\sqrt{429}}{5}$, score the marks.	
(b)	**B1** is for using 0.2 or $\frac{0.4}{2}$ as $\frac{1}{2}h$. **M1** requires first bracket to contain first plus last values and second bracket to include no additional values from those in the table. If the only mistake is to omit one value from 2nd bracket this may be regarded as a slip ar can be allowed (An extra repeated term forfeits the **M** mark however) x values: **M0** if values used in brackets are x values instead of y values. Separate trapezia may be used : **B1** for 0.2, **M1** for $\frac{1}{2}h(a+b)$ used 4 or 5 times (and **A1**ft all e.g.. $0.2(3+3.47)+0.2(3.47+3.84)+0.2(3.84+4.14)+0.2(4.14+4.58)$ is **M1 A0** equivalent to missing one term in { } in main scheme **A1ft** follows their answers to part (a) and is for {correct expression} Final **A1** must be correct. (No follow through)	
Special cases	Bracketing mistake: i.e. $\frac{1}{2}\times 0.4(3+4.58)+2(3.47+3.84+4.14+4.39)$ scores **B1 M1 A0 A0** <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). **Need to see trapezium rule – answer only (with no working) is 0/4.**	

The finite region R is bounded by the curve $y = 1 + 3\sqrt{x}$, the x-axis and the lines $x = 2$ and $x = 8$.

(a) Use the trapezium rule with three intervals of equal width to estimate to 3 significant figures the area of R. **(6)**

(b) Use integration to find the exact area of R in the form $a + b\sqrt{2}$. **(5)**

(c) Find the percentage error in the estimate made in part *(a)*. **(2)**

(a)	x	2	4	6	8
	$1 + 3\sqrt{x}$	5.243	7	8.348	9.485

M1 A1

area $\approx \frac{1}{2} \times 2 \times [5.243 + 9.485 + 2(7 + 8.348)]$ B1 M1 A1

$= 45.4$ (3sf) A1

(b) $= \int_2^8 (1 + 3\sqrt{x})\ \mathrm{d}x$

$= [x + 2x^{\frac{3}{2}}]_2^8$ M1 A1

$= [8 + 2(2\sqrt{2})^3] - [2 + 2(2\sqrt{2})]$ M1

$= (8 + 32\sqrt{2}) - (2 + 4\sqrt{2})$ M1

$= 6 + 28\sqrt{2}$ A1

(c) $= \frac{(6+28\sqrt{2})-45.4}{6+28\sqrt{2}} \times 100\% = 0.43\%$ M1 A1 **(13)**