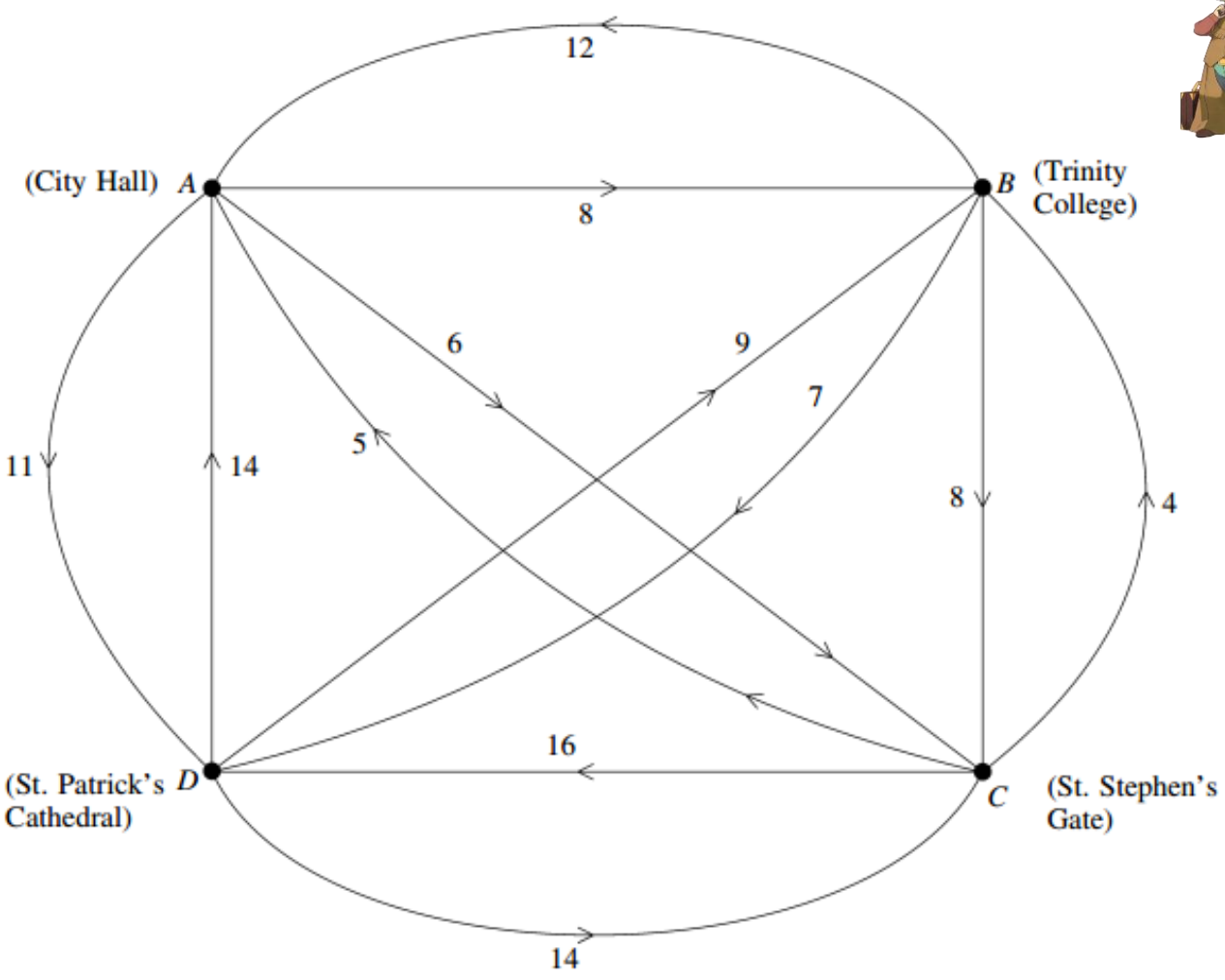
D1 Travelling Salesperson

Challenge 1

A family is staying in Dublin and is to visit four places of interest in the city. There is a one-way traffic system in the city and consequently the time taken to travel from *A* to *B* is different from the time taken to travel from *B* to *A*.

The following diagram shows the four places to be visited together with the time taken, in minutes, to travel between each two places.



(a) The family starts the tour at *A* and visits each of the other three places once before returning to *A*.

(i) Use the nearest neighbour algorithm to find an upper bound for the travelling time of the tour. *(4 marks)*

(ii) Find the number of different possible tours. *(2 marks)*

(b) Write down an expression for the number of possible tours if the family were to start at *A* and visit each of n other places once before returning to *A*. *(2 marks)*

Challenge 2

2 Roger, a football supporter, is to visit each of six football grounds. He decides to travel from one ground to the next until he has visited all of the grounds, starting and finishing at Man. City. The following table shows the distances, in miles, between the grounds.

	Man. City	Burnley	Crewe	Preston	Stockport	Tranmere
Man. City	–	19	26	32	8	31
Burnley	19	–	43	21	22	36
Crewe	26	43	–	42	19	23
Preston	32	21	42	–	36	26
Stockport	8	22	19	36	–	27
Tranmere	31	36	23	26	27	–

(a) Use the nearest neighbour algorithm, starting and finishing at Man. City, to find an upper bound for the total distance Roger must travel. *(4 marks)*

(b) By initially ignoring Man. City, find a lower bound for the total distance he must travel in visiting the six grounds. *(5 marks)*

(c) Using your answer to parts (a) and (b), write down inequalities for D, the total distance, in miles, that Roger has to travel. *(1 mark)*

Challenge 3

A sweet company has a production line making batches of seven different flavours of sweets A, B, C, D, E, F, G. The changeover times, in minutes, from the production line being set up for one flavour to it being set up for another flavour are given in the following table.

	A	B	C	D	E	F	G
A	-	13	17	18	16	15	14
B	13	-	19	16	18	17.5	16.5
C	17	19	-	24	22	21.5	22
D	18	16	24	-	23	22	21
E	16	18	22	23	-	20	19
F	15	17.5	21.5	22	20	-	18
G	14	16.5	22	21	19	18	-

(a) Normally sweets are produced in the order A, B, C, D, E, F, G. The production line is then set up to start with flavour A the next day.

(i) Find the total time taken up by the changeovers. *(2 marks)*

(ii) Explain why this answer can be considered to be an upper bound for this travelling salesman problem. *(2 marks)*

(b) Use the nearest neighbour algorithm, starting at A, to find a reduced time spent on changeovers. *(4 marks)*

(c) By initially ignoring sweet A, find a lower bound for the changeover times. *(5 marks)*

Final Challenge

6 A machine is used for producing sweets in six flavours. The machine produces one flavour of sweet at a time. It needs to be cleaned before changing flavours. The times taken to clean the machine depend on the two flavours involved and these times, in minutes, are given in the table below.

The machine is to be set to produce each flavour in sequence before repeating the cycle.

The machine can only start with Raspberry (*R*) or Strawberry (*S*).

		To					
		Blackcurrant (*B*)	**Lime** (*L*)	**Orange** (*O*)	**Plum** (*P*)	**Raspberry** (*R*)	**Strawberry** (*S*)
From	**Blackcurrant** (*B*)	–	25	20	20	27	25
	Lime (*L*)	15	–	10	11	15	30
	Orange (*O*)	5	30	–	15	20	19
	Plum (*P*)	20	25	15	–	25	10
	Raspberry (*R*)	10	20	7	15	–	15
	Strawberry (*S*)	15	25	19	10	20	–

(a) (i) Show that, using the nearest neighbour algorithm starting with Strawberry (*S*), the total cleaning time for one cycle is 85 minutes. *(3 marks)*

(ii) Use the same method starting with Raspberry (*R*) to find the total cleaning time for one cycle, which is less than 85 minutes. *(4 marks)*

(b) Explain why each of your answers to part (a) is an upper bound for the minimum **total** cleaning time for one cycle. *(2 marks)*

(c) Given that the machine produces *S* first followed by *R*, find an improved upper bound for the minimum total cleaning time for one cycle. *(5 marks)*