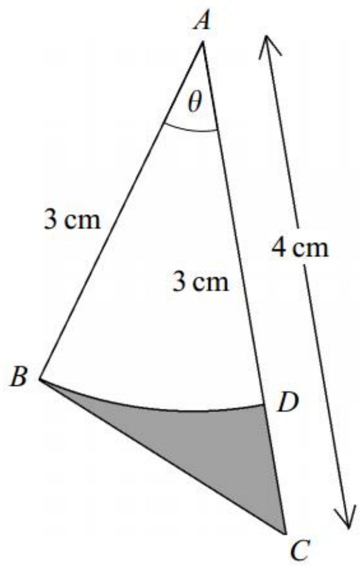
Triangles, Sectors and Arcs

The diagram shows a triangle ABC with $AB = 3$ cm, $AC = 4$ cm and angle $BAC = \theta$ radians.



The point D lies on AC such that $AD = 3$ cm, and ABD is a sector of a circle with centre A and radius 3 cm.

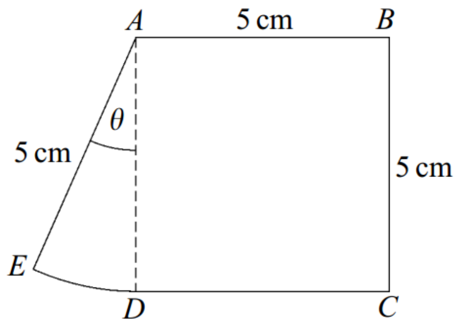
(a) Write down, in terms of θ:

(i) the area of the sector ABD; *(2 marks)*

(ii) the area of triangle ABC. *(2 marks)*

2(a)(i)	Area of sector $= \frac{1}{2}r^2\theta$	M1		For $\frac{1}{2}r^2\theta$
	$= 0.5 \times 9\theta = 4.5\theta \ (\text{cm}^2)$	A1	2	
(ii)	Area of triangle $= \frac{1}{2}AB \times AC \sin\theta$	M1		$\frac{1}{2}AB \times AC \sin\theta$
	$\ldots = \frac{1}{2}3 \times 4\sin\theta = 6\sin\theta \ (\text{cm}^2)$	A1	2	

The diagram shows a shape *ABCDE*. The shape consists of a square *ABCD*, with sides of length 5 cm, and a sector *ADE* of a circle with centre *A* and radius 5 cm. The angle of the sector is θ radians.



(a) Find the area of the sector *ADE* in terms of θ. *(2 marks)*

(b) The area of the sector *ADE* is a quarter of the area of the square *ABCD*.

(i) Find the value of θ. *(2 marks)*

(ii) Find the perimeter of the shape *ABCDE*. *(2 marks)*

3 (a)	Sector area formula stated Sector area = $12.5\ \theta$ (cm^2)	M1 A1	 2	or used Condone omission of units throughout
(b)(i)	Equating sector area to 6.25 $\theta = 0.5$	M1 A1	 2	
(ii)	Arc length formula stated Perimeter = 22.5 (cm)	M1 A1F	 2	or used ft wrong value of θ
	Total		**6**	

The acute angle θ radians is such that

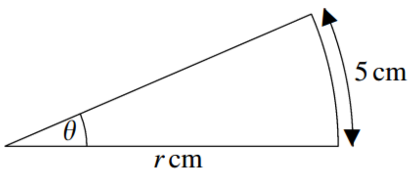
$$\sin\theta = \tfrac{5}{13}.$$

(a) (i) Show that $\cos\theta = \tfrac{12}{13}$. *(2 marks)*

(ii) Find the value of $\tan\theta$, giving your answer as a fraction. *(2 marks)*

(b) Use your calculator to find the value of θ, giving your answer to three decimal places. *(1 mark)*

(c) The diagram shows a sector of a circle of radius r cm and angle θ radians. The length of the arc which forms part of the boundary of the sector is 5 cm.



(i) Show that $r \approx 12.7$. *(2 marks)*

(ii) Find the area of the sector, giving your answer to the nearest square centimetre. *(3 marks)*

Q	Solution	Marks	Total	Comments
4 (a)(i)	Use of $\sin^2\theta + \cos^2\theta \equiv 1$	M1		OE, e.g. Pythagoras
	$\cos\theta = \frac{12}{13}$ convincingly shown	A1	2	AG but condone no mention of $\pm$
(ii)	Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$	M1		OE, eg right-angled triangle
	$\tan\theta = \frac{5}{12}$	A1	2	
(b)	$\theta \approx 0.395$	B1	1	Condone AWRT 0.395 or 22.6°
(c) (i)	Formula for arc length stated	M1		or used
	$r \approx \frac{5}{0.395} \approx 12.7$	A1	2	AG (12.7)
(ii)	Formula for sector area stated	M1		or used
	Substitution of appropriate values	m1		not $\frac{1}{2}(12.7^2)(22.6)$
	Area is $\frac{1}{2}(12.7)^2\ (0.395) \approx 32\ \text{cm}^2$	A1	3	Condone absence of units; accept AWRT 32
	Total		**10**	

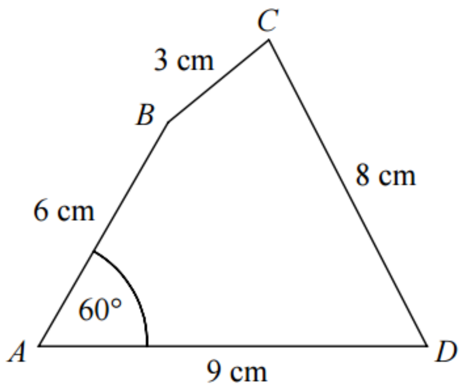


Figure 2

Figure 2 shows the quadrilateral $ABCD$ in which $AB = 6$ cm, $BC = 3$ cm, $CD = 8$ cm, $AD = 9$ cm and $\angle BAD = 60°$.

(a) Using the cosine rule, show that $BD = 3\sqrt{7}$ cm. **(4)**

(b) Find the size of $\angle BCD$ in degrees. **(3)**

(c) Find the area of quadrilateral $ABCD$. **(3)**

(a) $BD^2 = 6^2 + 9^2 - (2 \times 6 \times 9 \times \cos 60)$	M1 A1	
$BD^2 = 36 + 81 - 54 = 63$		
$BD = \sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$ cm	M1 A1	
(b) $(3\sqrt{7})^2 = 3^2 + 8^2 - (2 \times 3 \times 8 \times \cos C)$	M1	
$\cos C = \frac{9 + 64 - 63}{48} = \frac{5}{24}$		
$\angle BCD = 78.0°$ (1dp)	M1 A1	
(c) $= (\frac{1}{2} \times 6 \times 9 \times \sin 60) + (\frac{1}{2} \times 3 \times 8 \times \sin 77.975)$	M2	
$= 35.1$ cm^2 (3sf)	A1	**(10)**