7 (a) Find $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. *(2 marks)*

(b) The points P and Q lie on the lines

$$\mathbf{r} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \text{ respectively.}$$

(i) Express the vector $\overrightarrow{PQ}$ in terms of s and t. *(2 marks)*

(ii) Hence find the scalar λ for which

$$\overrightarrow{PQ} = \lambda\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

(4 marks)

(c) Show that the vector $\overrightarrow{PQ}$ with the value of λ found in part (b)(ii) is perpendicular to both lines. *(2 marks)*

(d) Find the shortest possible distance PQ. *(2 marks)*

Q	Solution	Marks	Total	Comments
7 (a)	$\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$	M1A1	2	
(b)(i)	$\overrightarrow{PQ} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - s\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$	M1A1	2	
(ii)	$= \lambda\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$	M1		
	Scalar product with $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$	M1		or equate components of vectors
	$(-2+9-2) = 14\lambda \quad \lambda = \frac{5}{14}$ or equation in λ	M1A1	4	
(c)	$\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ is $\perp$ to $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$	B1		
	and to $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$	B1	2	
(d)	$\frac{5}{14}\sqrt{14}$	M1A1F	2	
	Total		**12**	

8 The line l_1 has equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$.

The line l_2 has equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 2 \end{bmatrix} + s \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$.

(a) Show that the lines l_1 and l_2 intersect and find the coordinates of their point of intersection. *(5 marks)*

(b) (i) Show that the vector $\begin{bmatrix} 1 \\ 11 \\ -16 \end{bmatrix}$ is perpendicular to both l_1 and l_2. *(2 marks)*

(ii) Find the Cartesian equation of the plane containing the lines l_1 and l_2. *(3 marks)*

8 (a)	$3+4t=8-s$	M1		Set up and attempt to solve
	$-2+4t=-1+3s$			
	$t=1 \quad s=1$	m1A1		
	$1+3\times1=2+1\times2=4$	A1		Check third equation
	$(x,y,z)=(7,\ 2,\ 4)$	B1ft	5	ft on consistent use of s or t
(b)(i)	$4\times1+4\times11+3\times-16=0$	M1		Use scalar product with a direction
	$-1\times1+3\times11+2\times-16=0$	A1	2	Both equal zero
(ii)	Plane has equation $x+11y-16z=d$	M1		Use of normal vector
	$(3,\ -2,\ 1)$ is on the plane	M1		OE, use of a point on the plane
	$d=3+11\times-2-16\times1=-35$			or $\mathbf{n}\cdot\mathbf{r}=\mathbf{n}\cdot\mathbf{d}$
	$x+11y-16z=-35$	A1	3	
	Alternative to part (b)(ii) from vector equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$			
	$x=3+4\lambda-\mu$ $y=-2+4\lambda+3\mu$ $z=1+3\lambda+2\mu$	*(M1)* *(m1)* *(A1)*		Form simultaneous equations Complete elimination of λ,μ
	Total		**10**	

4 The planes Π_1 and Π_2 have equations

$$x + 2y - z = 1$$

and
$$x + 3y + z = 6$$

respectively.

(a) Verify that the point P, with coordinates $(1, 1, 2)$, lies on both planes. *(1 mark)*

(b) Find the equation of l, the line of intersection of Π_1 and Π_2, giving your answer in the form $\dfrac{x-\alpha}{p} = \dfrac{y-\beta}{q} = \dfrac{z-\gamma}{r}$. *(5 marks)*

(c) Find the acute angle between the line l and the line through P which is parallel to the y-axis. Give your answer to the nearest 0.1 of a degree. *(3 marks)*

4 (a)	$1+2-2=1, \quad 1+3+2=6$	B1	1	
(b)	$\begin{bmatrix}1\\2\\-1\end{bmatrix} \times \begin{bmatrix}1\\3\\1\end{bmatrix}$	M1A1		Alternative method for 4(b) Elimination of one letter e.g. $y = -2z + 5$ M1A1
	$= \begin{bmatrix}5\\-2\\1\end{bmatrix}$	A1F		Elimination of second letter e.g. $y = \dfrac{7-2x}{5}$ A1
	Equation of line is $\dfrac{x-1}{5} = \dfrac{y-1}{-2} = \dfrac{z-2}{1}$	M1A1F	5	Combining the results $-2z + 5 = y = \dfrac{7-2x}{5}$ M1 Rearranging $\dfrac{z-5/2}{1} = \dfrac{y}{-2} = \dfrac{x-7/2}{5}$ A1
(c)	$\cos\theta = \dfrac{\pm(0,1,0)\bullet(5,-2,\ 1)}{\sqrt{5^2+(-2)^2+1^2}}$	M1A1F		ft incorrect $(5, -2, 1)$
	$\theta = 68.6°$	A1F	3	
	Total		**9**	

5 A line l has equation

$$(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0},$$

where

$$\mathbf{a} = \begin{bmatrix} -4 \\ 10 \\ -4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}.$$

(a) (i) Show that the coordinates of any point on l may be written as

$$(-4 + 2\lambda,\ 10 - 3\lambda,\ -4 + \lambda),$$

where λ is a parameter. *(3 marks)*

(ii) Hence find the coordinates of the point N on l such that ON, where O is the origin, is perpendicular to l. *(4 marks)*

(b) Find the equation of the line through N which is perpendicular to both l and ON, giving your answer in the form $(\mathbf{r} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$. *(4 marks)*

Q	Solution	Marks	Total	Comments
5 (a)(i)	The direction of $\mathbf{r} - \mathbf{a}$ is parallel to $\mathbf{b}$	M1		
	$\mathbf{r} - \mathbf{a} = \lambda\mathbf{b}$	A1		
	$\mathbf{r} = \begin{bmatrix} -4 \\ 10 \\ -4 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -4+2\lambda \\ 10-3\lambda \\ -4+\lambda \end{bmatrix}$	A1	3	
(ii)	$\begin{bmatrix} -4+2\lambda \\ 10-3\lambda \\ -4+\lambda \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$	M1		
	$-8 + 4\lambda - 30 + 9\lambda - 4 + \lambda = 0$	A1		
	$\lambda = 3$	A1F		
	N is (2, 1, –1)	A1F	4	
(b)	$\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -8 \end{bmatrix}$	M1A1F		$\begin{bmatrix} -4 \\ 10 \\ -4 \end{bmatrix} \times \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ also earns M1
	$\mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{d} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ or any multiple	A1F A1F	4	
	Total		**11**	